

CHAPTER

09

Mechanical Properties of Fluids

Hydrostatics • Surface Tension • Hydrodynamics and Viscosity

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What's inside

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READING KEY

DEFINITION

NOTE

REMEMBER

DERIVATION

STRATEGY

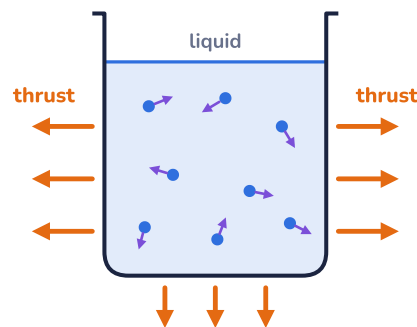
Formulas sit on graph paper • coloured words are the key terms

Fluids give **very little resistance** to a **shear stress**. Even a very small shear stress changes their shape. This is the **key property of fluids**. In this part we study fluids **at rest**.

1.1 Thrust

Take a fluid kept in a container. Its molecules are always in **random motion**. They move because of their **thermal velocities**.

So the molecules keep **hitting the walls** and bouncing back. In each hit, the momentum of a molecule changes in the direction **normal to the wall**. This normal momentum passes on to the wall.



The liquid pushes normally on the walls and the base

DEFINITION

The **thrust** of a fluid is the **normal momentum** that its molecules give to the container walls **in unit time**.

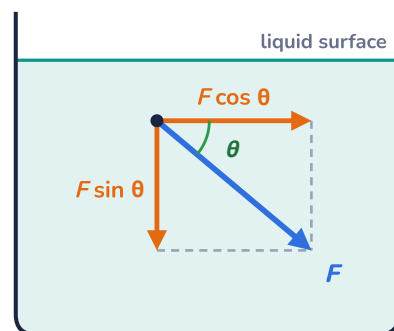
1.2 Liquid in Equilibrium

CONCEPT

For a liquid in **equilibrium**, the **net force** on it always acts **perpendicular to its surface**. The proof is given below.

SETUP – GIVEN INFORMATION

- ◆ A liquid is kept in a **vessel**. It is at **rest**.
- ◆ Suppose the net force F makes an **angle θ** with the direction **parallel to the surface**.
- ◆ Split F into **two components**, as in the figure.



F has a part $F \cos \theta$ along the surface and a part $F \sin \theta$ normal to it

i Tangential Component

$F \cos \theta$ acts **along (parallel to)** the liquid surface.

ii Normal Component

$F \sin \theta$ acts **perpendicular to** the liquid surface.

DERIVATION

The liquid is **at rest**, so it does not flow. Hence the part of the force along the surface must be **zero**:

1

$$F \cos \theta = 0 \quad \dots (i)$$

2

$$F \neq 0, \text{ so } \cos \theta = 0 \Rightarrow \theta = 90^\circ$$

RESULT

When a liquid is in **equilibrium at rest**, the net force always acts **perpendicular** to the liquid surface.

1.3 Pressure

DEFINITION

The **pressure** at a point in a liquid at rest is the **thrust (normal force)** that the liquid exerts **per unit area** around that point.

Let a total force F act **normally** on a flat area A . Then the pressure is:

PRESSURE

$$p = \frac{\text{thrust}}{\text{area}} = \frac{F}{A}$$

where, F = total normal force on the surface, A = area on which it acts

CGS Unit

dyne cm^{-2}

SI Unit

N m^{-2} (pascal)

Dimensions

$[\text{ML}^{-1}\text{T}^{-2}]$

GOOD TO KNOW

One Pascal (1 Pa)

A pressure of 1 N m^{-2} is called **1 pascal**. The unit is named after the French scientist **Blaise Pascal (1623–1662)**. He did the first major studies of pressure in fluids.

$$1 \text{ Pa} = 1 \text{ N m}^{-2} = 10 \text{ dyne cm}^{-2}$$

Atmosphere (atm)

The **atmosphere (atm)** is another common unit of pressure. **1 atm** is the pressure of the air **at sea level**.

$$1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$$

Force Not Spread Evenly

If the force is **not uniform**, the pressure is different at different points. Let a small normal force ΔF act on a small area ΔA around a point. Then:

$$p = \lim_{\Delta A \rightarrow 0} \frac{\Delta F}{\Delta A} = \frac{dF}{dA}$$

NOTE

Pressure is a Scalar

At a given point in a liquid, the pressure has the **same value in every direction**. So no definite direction goes with pressure. Hence pressure is a **scalar quantity**.

• PRACTICAL APPLICATIONS OF PRESSURE

For the same force, a **smaller area** gives a **larger pressure**.

i Knife

A **sharp knife** cuts more easily than a blunt knife. Its edge has a very small area, so the pressure is large.

ii Camel

A **camel** has broad feet. So it presses less on sand and walks easily. A man finds walking on sand difficult.

iii Railway Tracks

Tracks rest on **large wooden or iron sleepers**. They spread the weight and reduce the pressure on the ground.

iv Needle

A **sharp needle** goes into the skin easily, but a dull one does not. This happens even when we apply the **same force**.

GOOD TO KNOW

Ice-skating

The sharp metal edges of skates press hard on the ice. This pressure **melts** a little ice under them. The tiny water drops act like **rigid ball bearings**. So the skater glides smoothly.

1.4 Density and Relative Density

DEFINITION

The **density** of a substance is its **mass per unit volume**. It is written as ρ .

Let a substance have mass M and volume V . Then its density is:

DENSITY

$$\rho = \frac{M}{V}$$

where, M = mass of the substance, V = its volume

SI unit of density

kg m^{-3}

Dimensions of density

$[\text{ML}^{-3}]$

● RELATIVE DENSITY (SPECIFIC GRAVITY)

DEFINITION

The **relative density** (also called **specific gravity**) of a substance is its density **divided by** the **density of water at 4°C**.

RELATIVE DENSITY

$$\text{Relative density} = \frac{\text{Density of substance}}{\text{Density of water at 4°C}}$$

Unit and Nature

It is a ratio of two densities. So it has **neither a unit nor dimensions**. It is a **positive scalar**.

Water at 4°C

Water is **densest at 4°C**. Its density then is 1000 kg m^{-3} .

1.5 Pascal's Law

LAW

Pascal's law: In a fluid at rest, the pressure is the **same at all points** that are at the **same height**.

This was first noticed by the French scientist Blaise Pascal.

● PROOF OF PASCAL'S LAW

The proof uses **two ideas**:

i Normal Force

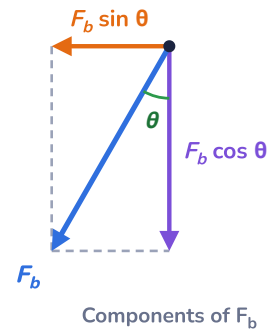
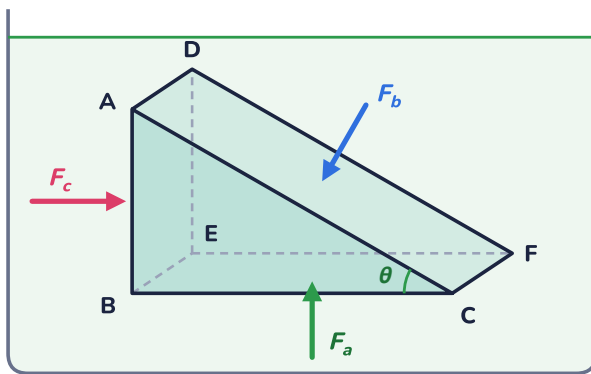
In a **fluid at rest**, the force on any layer is **normal to it**.

ii Newton's First Law

The small element of fluid is **in equilibrium**. So the forces on it balance.

SETUP – GIVEN INFORMATION

- ◆ Take a tiny element **ABC-DEF** inside a fluid at rest. It has the shape of a **right-angled prism**.
- ◆ The prism is so small that all its parts are at the **same depth**. So gravity has the same effect at all its points.
- ◆ Pressures p_a, p_b, p_c act on the faces **BEFC, ADFC and ADEB**.
- ◆ The normal forces on these faces are F_a, F_b, F_c . Their areas are A_a, A_b, A_c .



Tiny prism $ABC-DEF$ in a fluid at rest (left) and the two components of F_b (right)

DERIVATION

In the right triangle ABC , $\angle ACB = \theta$. Resolve the normal forces along the horizontal and the vertical:

1

$$F_b \sin \theta = F_c \quad \text{and} \quad F_b \cos \theta = F_a \quad \dots (i)$$

2

From the geometry of the prism, the face areas are related as:

$$A_b \sin \theta = A_c \quad \text{and} \quad A_b \cos \theta = A_a \quad \dots (ii)$$

3

Divide each force equation in (i) by the matching area equation in (ii):

$$\frac{F_b \sin \theta}{A_b \sin \theta} = \frac{F_c}{A_c} \quad \text{and} \quad \frac{F_b \cos \theta}{A_b \cos \theta} = \frac{F_a}{A_a}$$

4

$$\therefore \frac{F_a}{A_a} = \frac{F_b}{A_b} = \frac{F_c}{A_c}$$

RESULT OF PASCAL'S LAW

$$p_a = p_b = p_c$$

since $p = \frac{F}{A}$, all three faces feel the same pressure

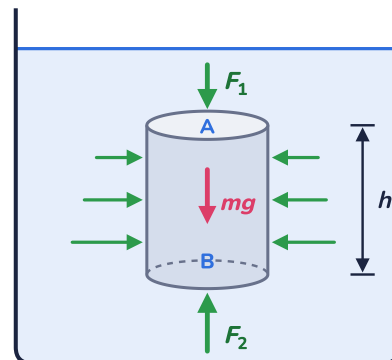
NOTE

So a fluid at rest presses on a body inside it **equally in all directions**. No direction can be given to this pressure. Hence pressure is **not a vector quantity**.

1.6 Variation of Pressure with Depth

SETUP – GIVEN INFORMATION

- ◆ A fluid of **density** ρ is at rest in a **cylindrical vessel**.
- ◆ Two points **A** and **B** are a **vertical distance** h apart.
- ◆ Imagine a cylinder of fluid with **cross-section area** A . Point A is on its top face and point B on its bottom face.



Imaginary fluid cylinder between A (top) and B (bottom)

DERIVATION

- 1 The weight of the fluid cylinder acts downward:

$$w = mg = \text{volume} \times \text{density} \times g = Ah\rho g \quad [\because V = A \times h]$$

- 2 The fluid is at rest. The horizontal forces cancel. So the total downward force equals the total upward force:

$$F_1 + w = F_2 \Rightarrow F_2 - F_1 = w$$

Here F_1 acts downward on the top face and F_2 acts upward on the bottom face.

- 3 Write each force as pressure $\times$ area:

$$p_2 A - p_1 A = Ah\rho g \Rightarrow A(p_2 - p_1) = A(h\rho g)$$

PRESSURE DIFFERENCE WITH DEPTH

$$p_2 - p_1 = h\rho g$$

NOTE

This pressure difference depends on three things: the **vertical height** between A and B, the **density** ρ of the fluid, and the value of g .

Now move point A up to the **free surface**, which is open to the air. Put $p_1 = p_a$ (atmospheric pressure) and $p_2 = p$:

PRESSURE AT A DEPTH h

$$p = p_a + h\rho g$$

At a depth h below an open liquid surface, the pressure is more than the atmospheric pressure by $h\rho g$.

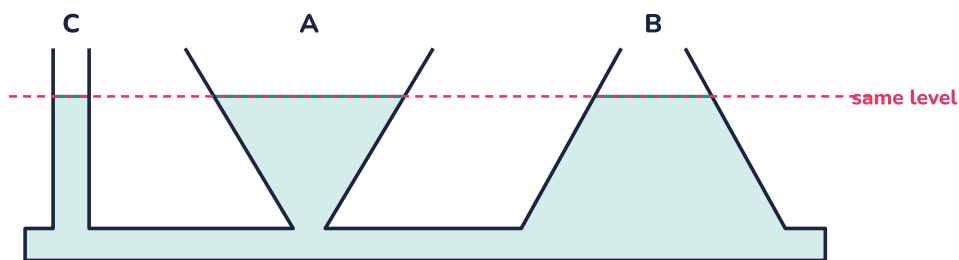
DEFINITION

The **gauge pressure** at a point is the **total pressure** there **minus the atmospheric pressure**.

$$p - p_a = h\rho g$$

1.7 Hydrostatic Paradox

Vessels **A**, **B** and **C** have **different shapes**. They are joined at the bottom. Each one holds a **different amount** of liquid. Yet the liquid level is the **same** in all three. This is called the **hydrostatic paradox**.



Vessels A, B and C hold different amounts of liquid, but the level is the same

CONCEPT

Hydrostatic Paradox

Liquid pressure depends **only on how tall the liquid column is**. The **shape** of the vessel does **not** matter.

Liquid pressure and the **height of the liquid column**

Depends on it

Liquid pressure and the **shape of the vessel**

Does not depend

1.8 Atmospheric Pressure and Gauge Pressure

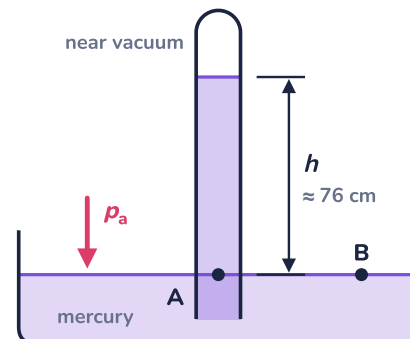
DEFINITION

The layer of air around the Earth is called the **Earth's atmosphere**. The pressure exerted by this air is called the **atmospheric pressure** (p_a). It equals the force of the air column on each **unit area** of the Earth's surface.

The atmosphere exerts a very large pressure. The methods below show this.

● MERCURY BAROMETER

- ◆ The Italian scientist **Evangelista Torricelli** (1608–1647) made the first instrument to measure atmospheric pressure. It is called a **barometer**.
- ◆ Fill a **long glass tube** (one end closed) with **mercury**. Then turn it upside down into a **trough of mercury**.
- ◆ The mercury in the tube settles about **76 cm** higher than the mercury in the trough.



Mercury barometer: A and B are at the same level

NOTE

Above the mercury column, the tube holds only **mercury vapour**. Its pressure is very small, so we **neglect** it.

Points A and B are at the same level, so their pressures are equal. So the atmospheric pressure equals the pressure of the mercury column of height h :

ATMOSPHERIC PRESSURE FROM A BAROMETER

$$p_a = h\rho g$$

where, h = height of the mercury column, ρ = density of mercury

EXAMPLE

Put $h = 0.76 \text{ m}$, $\rho = 13.6 \times 10^3 \text{ kg m}^{-3}$ and $g = 9.8 \text{ m s}^{-2}$:

$$p_a = 0.76 \times 13.6 \times 10^3 \times 9.8 = 1.013 \times 10^5 \text{ N m}^{-2} = 1.013 \times 10^5 \text{ Pa}$$

This value is the sea-level atmospheric pressure.

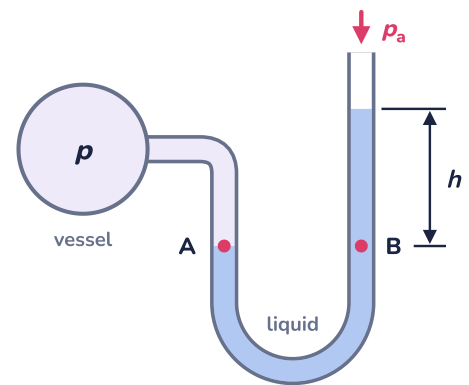
Pressure of a **1 mm column of mercury**

1 torr

● OPEN TUBE MANOMETER

We use an **open tube manometer** to measure **pressure differences**. The figure shows the set-up.

- ◆ It is a **U-shaped tube** with some liquid in it.
- ◆ One arm is **open to the air**. The other arm is joined to a vessel (flask) whose pressure we want to find.



Open tube manometer: A and B are at the same level

Low-density liquid (oil or water)

Small pressure differences

High-density liquid (mercury)

Large pressure differences

NOTE

Pressure Relation

Points A and B are at the same level in the same liquid. So by **Pascal's law**, the pressure p at A equals the pressure at B:

$$p = p_A = p_B = p_a + h\rho g$$

• ABSOLUTE PRESSURE AND GAUGE PRESSURE

Absolute Pressure

Absolute pressure is the full, **actual pressure** p at a point.

Gauge Pressure

The **gauge pressure** is the absolute pressure at a point **minus the atmospheric pressure**.

GAUGE PRESSURE

$$p_g = p - p_a = h\rho g$$

So the gauge pressure is **proportional to** h .

GOOD TO KNOW

Many pressure gauges **read the gauge pressure directly**. Examples: a gauge that checks **tyre pressure**, and a **blood pressure gauge (sphygmomanometer)**.

• HEIGHT OF THE ATMOSPHERE

Standard atmospheric pressure is 1.013×10^5 Pa. Suppose the air had a **uniform density** $\rho = 1.29 \text{ kg m}^{-3}$ all the way up. Let h be the height of the air column that gives this pressure. Then:

$$h\rho g = 1.013 \times 10^5 \Rightarrow h = \frac{1.013 \times 10^5}{\rho g} = \frac{1.013 \times 10^5}{1.29 \times 9.8} \text{ m}$$

HEIGHT OF THE ATMOSPHERE (UNIFORM DENSITY)

$$h \approx 8.01 \times 10^3 \text{ m} \approx 8 \text{ km}$$

NOTE

In reality, the **density of air falls with height**. The value of g also falls. In fact, the atmosphere reaches a height of nearly **100 km**.

• DIFFERENT UNITS OF PRESSURE

i Atmosphere (atm)

It equals the pressure of a mercury column **76 cm** high, at 0°C , at 45° latitude and at mean sea level.

ii Bar and Millibar

Used in **meteorology** (weather science).

$$1 \text{ bar} = 10^5 \text{ Pa} = 10^6 \text{ dyne cm}^{-2}$$

$$1 \text{ mbar} = 10^{-3} \text{ bar} = 100 \text{ Pa}$$

iii Torr

Named after **Torricelli**.

$$1 \text{ torr} = 1 \text{ mm of Hg} = 133 \text{ Pa}$$

UNIT RELATION

$$1 \text{ atm} = 1.013 \text{ bar} = 760 \text{ torr}$$

NOTE

- ◆ If the atmospheric pressure falls by **10 mm of Hg or more**, a **storm** is likely on its way.
- ◆ Gauge pressure can be **positive** ($p > p_a$) or **negative** ($p < p_a$).
- ◆ In **inflated tyres** and in the **human circulatory system**, the absolute pressure is more than the atmospheric pressure. So the gauge pressure is positive. It is called **over pressure**.
- ◆ When we **suck a drink through a straw**, the pressure in our lungs is less than the atmospheric pressure. So the gauge pressure there is below zero.

Tyres and blood vessels ($p > p_a$)

Positive gauge pressure

Drinking with a straw ($p < p_a$)

Negative gauge pressure

1.9 Pascal's Law for Transmission of Fluid Pressure

LAW

Pascal's law (transmission of pressure): Apply an **external pressure** to any part of a fluid that is enclosed in a vessel and at rest. The fluid passes this pressure on to every part of it. The pressure reaches every point **without any loss** and acts **equally in all directions**.

Pressure applied to one part of a confined fluid

Passed equally in all directions

NOTE

This law has **many uses in daily life**. Some of them are given in the next topic.

1.10 Applications of Pascal's Law

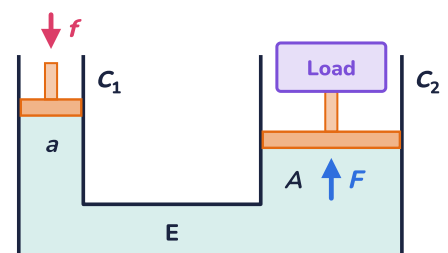
• (I) HYDRAULIC LIFT

CONCEPT

A **hydraulic lift** is used to **raise heavy loads**. It is based on Pascal's law and works as a **force multiplier**.

SETUP – GIVEN INFORMATION

- ◆ Two cylinders C_1 and C_2 are joined by a **pipe E**.
- ◆ Each has a **tight, frictionless piston**. The piston areas are **different**.
- ◆ A **liquid** fills the cylinders and the pipe.



Hydraulic lift: f on area a , F on area A

DERIVATION

- 1 Push the **smaller piston** (area a) down with a force f . The pressure on the liquid is:

$$p = \frac{f}{a}$$

- 2 By **Pascal's law**, this pressure reaches the **larger piston** (area A). So the upward force on it is:

$$F = p \times A = \frac{f}{a} \times A = \frac{A}{a} \times f$$

UPWARD FORCE ON THE LARGER PISTON

$$F = \frac{A}{a} f$$

Since $A > a$, we get $F > f$. A small force gives a large force

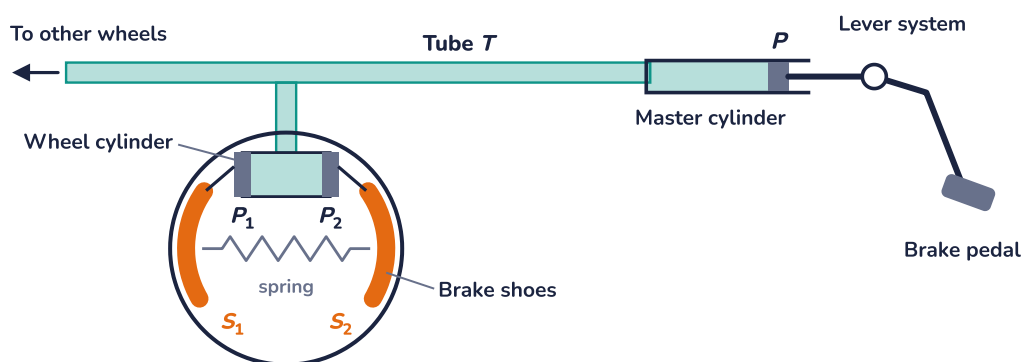
Heavy load lifted

• (II) HYDRAULIC BRAKES

Hydraulic brakes also work on **Pascal's law**.

Construction

- ◆ A **tube T** is filled with **brake oil**.
- ◆ One end of T goes to the **wheel cylinder**. It has two pistons, P_1 and P_2 . These pistons are joined to the **brake shoes** S_1 and S_2 .
- ◆ The other end of T goes to the **master cylinder**. A lever system links its piston **P** to the **brake pedal**.



Hydraulic brakes: master cylinder (P), tube T , wheel cylinder (P_1, P_2) and brake shoes (S_1, S_2)

Cross-section area: **wheel cylinder** compared with **master cylinder**

Wheel cylinder is larger

• WORKING OF HYDRAULIC BRAKES

- ◆ **Pedal pressed:** When the driver **presses the brake pedal**, the lever system pushes piston **P**. By **Pascal's law**, the pressure reaches pistons P_1 and P_2 .

- ◆ **Shoes pushed out:** Pistons P_1 and P_2 push the **brake shoes apart**. The shoes press on the **inner rim of the wheel** and **slow down** the wheel.
- ◆ **Force multiplied:** The wheel-cylinder pistons have a **larger area** than piston P. So a **small force** on the pedal gives a **large thrust** on the wheels. This makes the brakes work.
- ◆ **Pedal released:** When the driver **releases the pedal**, a **spring** pulls the shoes back from the rim to their normal place. This pushes the oil **back into the master cylinder**.
- ◆ **All wheels:** The master cylinder is joined by tubes to **all the wheels**. So **equal pressure** reaches every wheel.
- ◆ **Result:** So a **small force** on the pedal puts a **far larger force** on the drums of the wheels. This helps the driver keep the vehicle **under control**.

1.11 Buoyancy

DEFINITION

When a body is **partly or fully immersed** in a fluid at rest, it pushes aside (**displaces**) some fluid. The fluid then pushes the body **upward**.

This upward force on the body is called the **upthrust** or **buoyant force**. The effect is called **buoyancy**.

Where the Weight Acts

The **weight** acts at the **centre of gravity** of the body.

Where the Upthrust Acts

The **buoyant force** acts through the **centre of buoyancy**. This point is the centre of gravity of the displaced liquid.

1.12 Archimedes' Principle

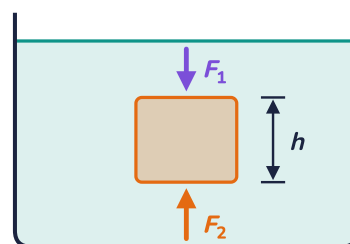
PRINCIPLE

Archimedes' principle gives the size of the buoyant force. A body fully or partly immersed in a liquid at rest feels an **upthrust**. The upthrust equals the **weight of the liquid** that the immersed part of the body **pushes aside (displaces)**. This force passes through the **centre of gravity of the fluid displaced**.

• PROOF OF ARCHIMEDES' PRINCIPLE

SETUP – GIVEN INFORMATION

- ◆ A **cylinder** of height h and area A is immersed in a fluid.
- ◆ Upward force F_2 on its bottom is **larger** than downward force F_1 on its top.
- ◆ p_1, p_2 = pressures on the top and bottom faces.



Forces on an immersed body

DERIVATION

1 The net upward force of the fluid on the body is:

$$F_2 - F_1 = (p_2 - p_1)A$$

2 From the pressure difference with depth:

$$p_2 - p_1 = h\rho g$$

3 So the net upward force is:

$$(p_2 - p_1)A = h\rho gA = mg \quad [\because m = A\rho h]$$

Here ρ = density of the fluid and A = cross-section area of the body. So m is the mass of the fluid displaced.

RESULT

The fluid pushes the body up with a force equal to the **weight of the fluid it displaces**, that is, mg .

LAWS OF FLOATATION

A body placed in a fluid can behave in **three ways**. Let W_b be its weight and F_B the upthrust when it is fully immersed.

i Weight > Upthrust

$W_b > F_B$: the body **sinks**.

ii Weight = Upthrust

$W_b = F_B$: the weight is just balanced. The body **floats fully immersed**.

iii Weight < Upthrust

$W_b < F_B$: the body **floats partly immersed**.

LAW

Law of Floatation

Let a floating body have total volume V_b . Let a part V_p of it be under the fluid. At equilibrium, the **weight of the body equals the weight of the fluid it displaces**:

$$V_b \rho_b g = V_p \rho_f g$$

LAW OF FLOATATION

$$\frac{\rho_b}{\rho_f} = \frac{V_p}{V_b}$$

where, ρ_b = density of the body, ρ_f = density of the fluid

02 PART 2 Surface Tension

2.1 Surface Tension

CONCEPT

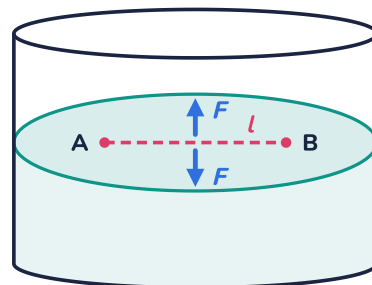
A liquid at rest has some **extra potential energy** at its free surface. Because of this, the surface tries to keep its area as **small as possible**. So the surface acts like a **stretched membrane**.

e.g. Steel is denser than water. Still, a **steel needle** can be made to float on water. The water surface acts like a stretched elastic sheet and holds the needle up. This property is called **surface tension**.

Imagine a line **AB** drawn on a liquid surface. The surface on each side pulls this line. The two pulls are **equal and opposite**. They act **perpendicular to the line**. So the line stays in equilibrium.

The pull is **proportional to the length** of the line. Let l be its length and F the force on either side. Then:

$$F \propto l \Rightarrow F = Sl \Rightarrow S = \frac{F}{l}$$



Equal and opposite pulls F on the line AB

DEFINITION

Surface tension (S) is the **force per unit length** on an imaginary line drawn on the free surface of a liquid. The force acts **at right angles to the line** and lies along the surface (it is **tangential**). Surface tension is a **scalar quantity**.

SURFACE TENSION

$$S = \frac{F}{l} = \frac{\text{Force}}{\text{Length}}$$

SI Unit

$$\text{N m}^{-1}$$

CGS Unit

$$\text{dyne cm}^{-1}$$

Dimensions

$$\frac{[\text{MLT}^{-2}]}{[\text{L}]} = [\text{ML}^0\text{T}^{-2}]$$

• FACTORS AFFECTING SURFACE TENSION

1 • TEMPERATURE

Surface tension **falls** as the temperature rises. It becomes **zero** at a certain temperature. This is called the **critical temperature** of the liquid.

For small changes in temperature, it falls almost linearly: $S_t = S_0(1 - \alpha t)$

Here S_t = surface tension at $t^\circ\text{C}$, S_0 = surface tension at 0°C and α = temperature coefficient of surface tension.

- ◆ Soup tastes nicer **when hot** than when cold. Hot soup has a lower surface tension, so it spreads over more of the tongue.

2 • IMPURITIES

Adding impurities changes surface tension a lot.

- ◆ **Sodium chloride** dissolves easily (it is **highly soluble**). Adding it **raises** the surface tension of water.
- ◆ **Phenol or soap** dissolves only a little (**sparingly soluble**). Adding it **lowers** the surface tension of water.

• DETERGENTS AND SURFACE TENSION

- ◆ **Detergents** used to wash dirty clothes show a common use of surface tension.
- ◆ Oily dirt on clothes does not get wet by plain water. So water by itself **cannot remove the dirt**.
- ◆ A detergent molecule **sticks to both** water and dirt molecules. So washing carries the dirt away.

• APPLICATIONS OF SURFACE TENSION

i Round Drops

Rain drops, and drops of **mercury** on a glass plate, are **spherical**.

ii Brush Hair

The hair of a **shaving or painting brush** spread out in water. They **cling together** as soon as the brush is taken out.

iii Floating Needle

A **greased needle**, laid gently on water, **stays afloat**.

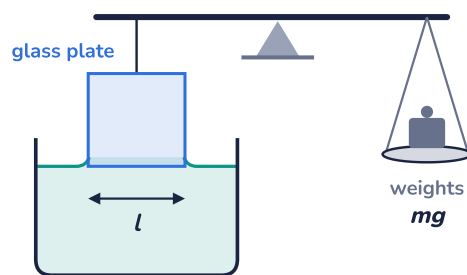
iv Oil Drops

An **oil drop** spreads out on cold water. On hot water, it keeps its shape.

• MEASURING THE SURFACE TENSION OF A LIQUID

A thin, flat glass plate is held **vertical**. It hangs from one arm of a balance. Weights on the other arm balance it. A vessel of the liquid is kept below the plate.

The vessel is raised until the liquid just touches the plate. Surface tension then **pulls the plate down**. Now **weights are added** until the plate just leaves the liquid. Let this extra weight be mg . The liquid touches both faces of the plate edge, so the total length is $2l$.



A glass plate pulled down by surface tension, balanced by weights

$$S = \frac{mg}{2l}$$

where, m = extra mass needed, l = length of the plate edge

2.2 Surface Energy

DEFINITION

A liquid surface always tries to **shrink** to the smallest possible area. To increase this area at constant temperature, **work must be done**. This work is stored in the surface film as **potential energy**.

The **surface energy** is this potential energy stored per unit area of the surface film. In other words, it is the **work needed to increase the surface area by one unit**.

SURFACE ENERGY

$$\text{Surface energy} = \frac{\text{Work done in increasing the surface area}}{\text{Increase in surface area}}$$

SI unit of surface energy: J m^{-2}

• HOW SURFACE ENERGY RELATES TO SURFACE TENSION

SETUP – GIVEN INFORMATION

- ◆ Take a **rectangular wire frame PQRS**. Its side **QR can slide**.
- ◆ Dip the frame in soap solution to form a **soap film** on it. Surface tension pulls the wire QR **inward**.
- ◆ QR is pulled outward through a small distance against this force. This stretches the film.

DERIVATION

- 1 Surface tension is force per unit length. A soap film has **two surfaces**, so the total length on QR is $2l$:

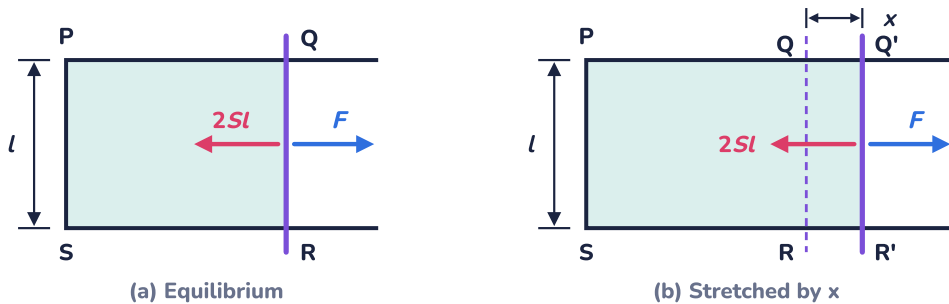
$$S = \frac{F'}{2l} \Rightarrow F' = S \times 2l$$

- 2 An outside force F (equal to F') moves QR very slowly through a distance x . The work done is:

$$W = \text{Force} \times \text{distance} = S \times 2l \times x$$

- 3 The film has two sides, so its area grows by $2l \times x$. Hence:

$$\text{Surface energy} = \frac{\text{Work done}}{\text{Surface area}} = \frac{S \cdot 2lx}{2lx} = S$$



(a) Equilibrium
Soap film on frame $PQRS$: (a) in equilibrium, (b) after QR is pulled out by x

SURFACE ENERGY AND SURFACE TENSION

$$\text{Surface energy} = S$$

Surface energy and surface tension of a liquid have the **same numerical value**.

2.3 Angle of Contact

DEFINITION

Near the place where a liquid touches another medium, the liquid surface is **curved**. The **angle of contact** (θ) is the angle that the **tangent to the liquid surface**, drawn at the point of contact, makes with the **solid surface** inside the liquid.

The angle of contact depends on these **factors**:

i Materials

The nature of the **solid and the liquid** in contact.

ii Cleanliness

How **clean** the surfaces in contact are.

iii Medium

The **medium** (like air) above the liquid surface.

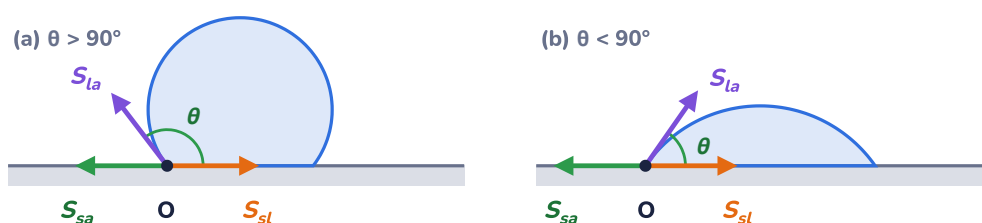
iv Temperature

How hot the liquid is (its **temperature**).

e.g. Water forms **drops on a lotus leaf**. But it **spreads out on a clean plastic plate**.

Three surfaces meet at the line of contact. Each pair has its own surface tension:

- ◆ S_{sa} : surface tension between the **solid and air**
- ◆ S_{la} : surface tension between the **liquid and air**
- ◆ S_{sl} : surface tension between the **solid and liquid**



Drops with (a) an obtuse and (b) an acute angle of contact θ

At the line of contact, the three surface forces are in **equilibrium**. Split S_{la} into two parts: $S_{la} \cos \theta$ along the solid surface and $S_{la} \sin \theta$ perpendicular to it. For balance along the solid, we get:

ANGLE OF CONTACT

$$S_{sa} = S_{sl} + S_{la} \cos \theta \quad \Rightarrow \quad \cos \theta = \frac{S_{sa} - S_{sl}}{S_{la}}$$

So there are **two cases**:

i $S_{sl} > S_{sa}$

$\cos \theta$ is **negative**, so $\theta > 90^\circ$ (**obtuse**). Such liquids **do not wet** the solid.

e.g. Mercury on glass: $\theta > 90^\circ$, non-wetting.

ii $S_{sl} < S_{sa}$

$\cos \theta$ is **positive**, so $\theta < 90^\circ$ (**acute**). Such liquids **wet** the solid.

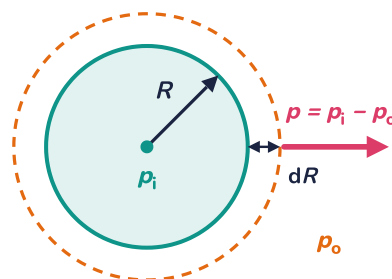
e.g. Water on glass: $\theta < 90^\circ$, wetting.

2.4 Excess Pressure Inside Drops and Bubbles

• EXCESS PRESSURE: LIQUID DROP

SETUP – GIVEN INFORMATION

- Take a liquid drop of radius R . It is a **sphere**. Its surface tension is S .
- Because it is curved, the pressure inside is **more** than outside. This **excess pressure** p acts normally outward.
- $p = p_i - p_o$, where p_i = pressure inside and p_o = atmospheric pressure outside.
- Let this pressure increase the radius from R to $R + dR$.



Liquid drop: excess pressure $p = p_i - p_o$

DERIVATION

1 First surface area = $4\pi R^2$. When the radius becomes $R + dR$, the new area is:

$$4\pi(R + dR)^2 = 4\pi R^2 + 8\pi R dR \quad [dR^2 \text{ neglected}]$$

2 So the area grows by $8\pi R dR$. Work done = gain in surface energy = (gain in area) $\times$ surface tension:

$$W = (8\pi R dR) S \quad \dots (i)$$

3 This work is also done by the excess pressure. Work = excess pressure $\times$ area $\times$ change in radius:

$$W = p \times 4\pi R^2 \times dR \quad \dots (ii)$$

4 Equate (i) and (ii):

$$p \times 4\pi R^2 \times dR = 8\pi R dR S$$

EXCESS PRESSURE INSIDE A LIQUID DROP

$$p_i - p_o = \frac{2S}{R}$$

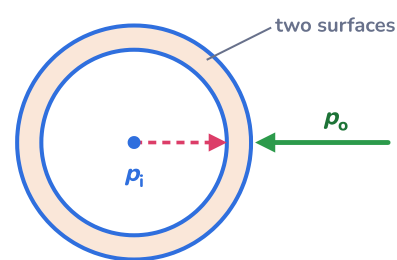
● EXCESS PRESSURE: SOAP BUBBLE

A soap bubble is a thin film. It has **two free surfaces**, one inside and one outside. So the real gain in surface area is doubled:

$$\text{Total gain in area} = 2 \times 8\pi R dR = 16\pi R dR$$

Equate the work done to the gain in surface energy:

$$p \times 4\pi R^2 \times dR = 16\pi R dR S$$



Soap bubble: a thin film with two surfaces

EXCESS PRESSURE INSIDE A SOAP BUBBLE

$$p_i - p_o = \frac{4S}{R}$$

● AIR BUBBLE INSIDE A LIQUID

Inside a liquid, an air bubble behaves like a liquid drop in air. It has just **one free surface**. So the extra pressure inside it is:

EXCESS PRESSURE INSIDE AN AIR BUBBLE

$$p = \frac{2S}{R}$$

NOTE

Let an air bubble of radius R be at a **depth** h below the free surface of a liquid. The liquid has density ρ and surface tension S . Then the pressure inside the bubble is more than the atmospheric pressure by:

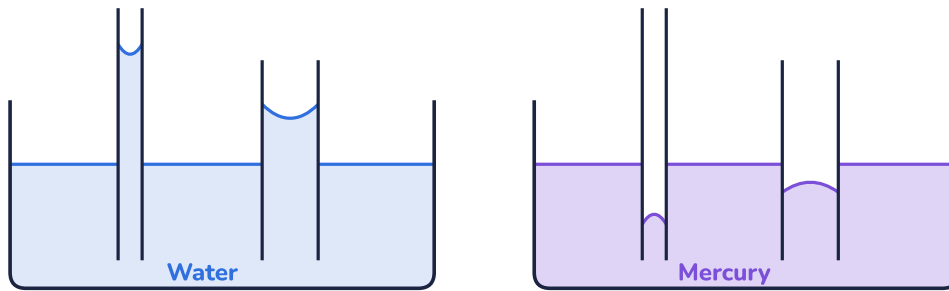
$$p = \frac{2S}{R} + h\rho g$$

2.5 Capillarity

DEFINITION

Capilla is a Latin word. It means **hair**. A tube with a very fine bore, as thin as a hair, is called a **capillary tube**.

Dip a glass capillary in **water**: the water **rises** in it. Dip it in **mercury**: the mercury level **falls** in it. This rise or fall of a liquid in a capillary tube is called **capillarity**.

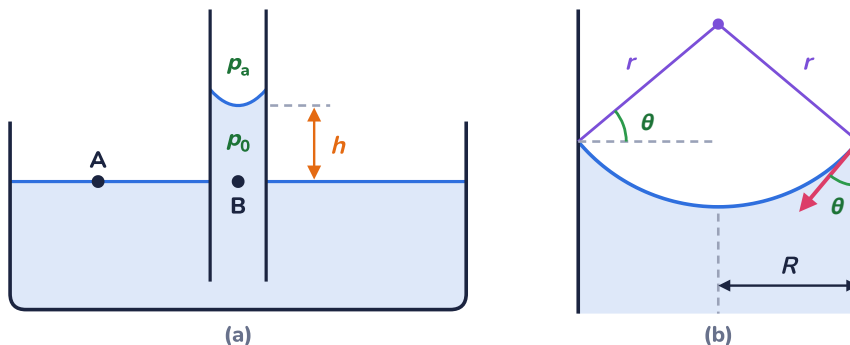


Water rises in glass tubes (concave surface). Mercury falls (convex surface)

Some Examples

- ◆ We use **towels** to dry our skin.
- ◆ In trees, **sap rises** through fine vessels. This is like capillary action.

● CAPILLARY RISE



(a) Water rises by h in a capillary. (b) Concave surface: radius of curvature r , tube radius R , angle of contact θ

SETUP – GIVEN INFORMATION

- ◆ A pressure difference exists across a curved liquid surface. One result of this is that water **rises in a narrow tube (capillary)** against gravity.
- ◆ A capillary of radius R is dipped in a vessel of water.
- ◆ The water surface in the capillary becomes **concave**. So there is a **pressure difference** across the two sides of this surface (meniscus).

DERIVATION

- Let r be the radius of curvature of the concave surface. From figure (b), $r = R \sec \theta$. The pressure p_a on the air side is more than the pressure p_0 just below the surface:

$$(p_a - p_0) = \frac{2S}{r} = \frac{2S}{R \sec \theta} = \frac{2S}{R} \cos \theta \quad \dots (i)$$

- Points A and B are at the same level. By Pascal's law, they have the **same pressure**. So $p_0 + \rho gh = p_a$, which gives:

$$p_a - p_0 = \rho gh \quad \dots (ii) \quad [p_a = \text{atmospheric pressure}]$$

- From equations (i) and (ii):

$$\rho gh = \frac{2S}{R} \cos \theta$$

HEIGHT OF LIQUID RISE IN A CAPILLARY

$$h = \frac{2S \cos \theta}{\rho R g}$$

Liquids that **wet** glass (like water) **rise** in the capillary. Liquids that **do not wet** glass **fall** in it.

Blotting paper soaks up ink

Capillarity

A towel soaks up water

Capillary action

03 PART 3 Hydrodynamics and Viscosity

The study of **moving fluids** is called **hydrodynamics** or **fluid dynamics**.

3.1 Streamline, Laminar and Turbulent Flow

• STREAMLINE FLOW

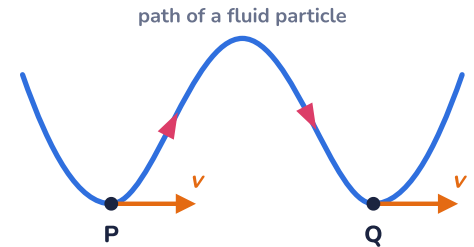
DEFINITION

In **streamline flow**, every particle that reaches a given point moves exactly like the particle that passed there before it. It follows the **same path** and has the **same velocity**.

DEFINITION

A **streamline** is a curve drawn in the flow. Its **tangent at any point** shows the direction of the fluid's velocity there.

A streamline is the path of a fluid particle in steady flow.



Velocity is along the tangent at every point

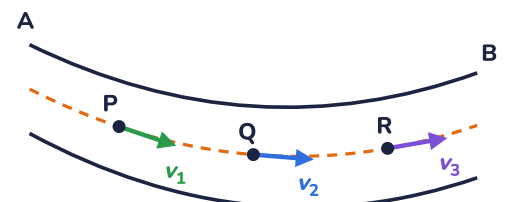
• PROPERTIES OF STREAMLINES

i They Never Cross

Two streamlines can **never cross**. At a crossing point, a particle would have two directions of motion at once. Then the flow would not be steady.

ii Crowding Means Speed

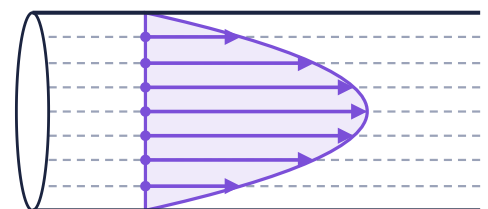
Where the streamlines come **closer together**, the liquid moves **faster**. Where they are spread out, it moves slower.



A region of streamline flow

• LAMINAR FLOW

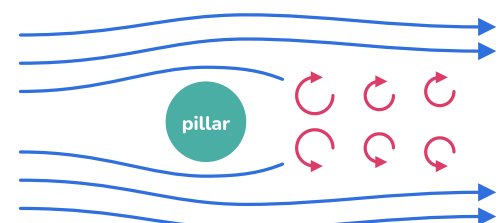
Sometimes a liquid flows over a horizontal surface as **layers moving with different velocities**. Such a flow is called **laminar flow**. No particle leaves its own layer to **move into another**. Laminar flow is usually a streamline flow.



Laminar flow: layers slide with different speeds

• TURBULENT FLOW

In rivers and canals, the water may move **very fast**. The boundary surfaces may also cause **sudden changes in velocity**. Then the flow becomes **irregular and disordered**. Such a flow is called **turbulent flow**.



Turbulent flow: eddies form behind a pillar

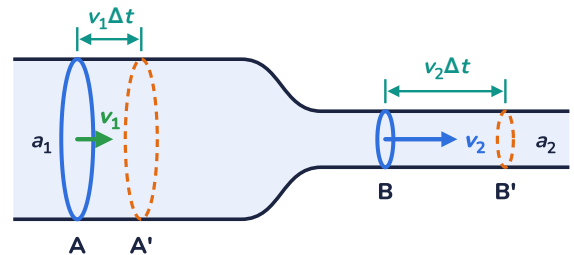
3.2 Equation of Continuity

LAW

Equation of continuity: Let a **non-viscous, incompressible** fluid flow in a streamline way through a pipe whose **cross-section changes**. Then the product of the cross-section area and the fluid velocity (av) is the **same at every point** of the pipe.

SETUP – GIVEN INFORMATION

- ◆ A liquid of density ρ flows through a tube **AB**. The liquid is non-viscous and incompressible. The cross-section of AB changes along its length.
- ◆ At **A**: area a_1 , velocity v_1 .
- ◆ At **B**: area a_2 , velocity v_2 .



Fluid flow through pipe AB

DERIVATION

- 1 In time Δt , the mass of fluid entering at A and the mass leaving at B are:

$$m_1 = a_1 v_1 \Delta t \rho \quad \text{and} \quad m_2 = a_2 v_2 \Delta t \rho$$

- 2 By **conservation of mass**, $m_1 = m_2$. The fluid is incompressible, so ρ stays the same:

$$a_1 v_1 \Delta t \rho = a_2 v_2 \Delta t \rho \Rightarrow a_1 v_1 = a_2 v_2$$

EQUATION OF CONTINUITY

$$a_1 v_1 = a_2 v_2 = av = \text{constant}$$

In a **narrow** part of the tube the fluid **speeds up**. In a **wide** part it **slows down**.

3.3 Bernoulli's Theorem

PRINCIPLE

Bernoulli's theorem: Take an **incompressible, non-viscous** fluid in **streamline, irrotational** flow. The total of its **pressure energy, kinetic energy and potential energy, each per unit volume**, is the **same** at every cross-section along the flow.

The theorem comes from the law of conservation of energy.

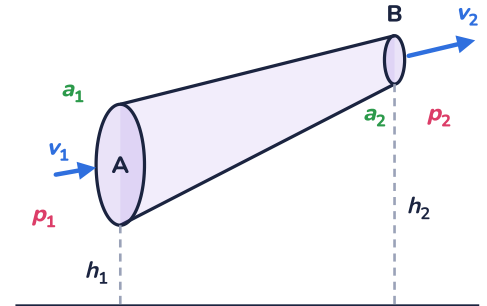
BERNOULLI'S EQUATION

$$p + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$$

p = pressure energy, $\frac{1}{2}\rho v^2$ = kinetic energy, ρgh = potential energy, all **per unit volume** (ρ = density). **Daniel Bernoulli** gave it in 1738.

SETUP – GIVEN INFORMATION

- The fluid is **ideal**, that is, non-viscous and incompressible. It has **streamline flow** through a pipe whose **cross-section changes**.
- At A: pressure p_1 , area a_1 , velocity v_1 , height h_1 .
- At B: pressure p_2 , area a_2 , velocity v_2 , height h_2 .



An ideal fluid flowing through a pipe whose cross-section changes

DERIVATION

- The pressure forces do net work on the fluid. Using $a_1 v_1 = a_2 v_2$, this work is:

$$W = \left(\frac{p_1}{\rho} - \frac{p_2}{\rho} \right) m$$

- This work increases the kinetic and potential energy of the fluid. The work-energy theorem then gives:

$$\frac{p_1}{\rho} + \frac{1}{2}v_1^2 + gh_1 = \frac{p_2}{\rho} + \frac{1}{2}v_2^2 + gh_2$$

PER UNIT VOLUME

$$p + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$$

IN TERMS OF HEADS (÷ ρg)

$$\frac{p}{\rho g} + \frac{v^2}{2g} + h = \text{constant}$$

Here $\frac{p}{\rho g}$ is the **pressure head**, $\frac{v^2}{2g}$ is the **velocity head** and h is the **gravitational head**.

NOTE

Bernoulli's Equation for a Fluid at Rest

For a fluid at rest, $v_1 = v_2 = 0$. So the velocity terms drop out:

$$p_1 - p_2 = \rho g(h_2 - h_1)$$

• LIMITATIONS OF BERNOULLI'S THEOREM

- ◆ **Zero viscosity:** it strictly holds only for non-viscous fluids.
- ◆ **Incompressible:** the fluid must be incompressible. Its elastic energy is not counted.
- ◆ **Streamline only:** it is **not valid** when the flow is turbulent or unsteady.

3.4 Applications of Bernoulli's Theorem

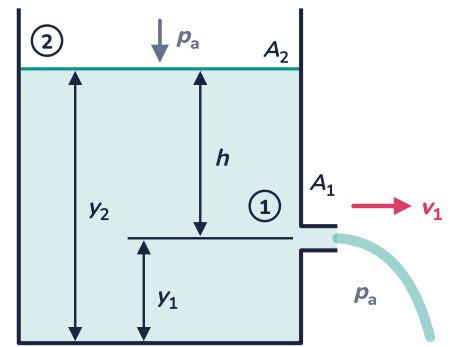
• TORRICELLI'S LAW: SPEED OF EFFLUX

The **speed of efflux** is the speed of a liquid coming out of a small hole (**orifice**) in a tank. The orifice is at depth h below the free surface. Bernoulli's equation for the surface and the orifice gives:

TORRICELLI'S LAW

$$v = \sqrt{2gh}$$

Same speed as a body falling freely through height h .



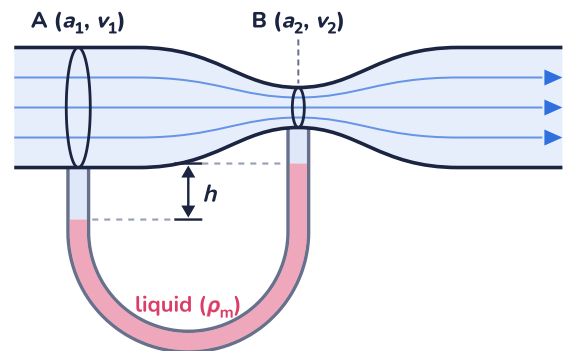
Liquid coming out of an orifice at depth h

• VENTURIMETER

A **venturimeter** (venturi tube) **measures the flow speed** of an incompressible fluid in a pipe.

This horizontal tube has a **wide part** (area a_1) and a **narrow neck** (area a_2). A manometer joins them. Continuity and Bernoulli's equation link the speed to the pressure difference:

$$p_1 - p_2 = \frac{1}{2} \rho v_1^2 \left(\frac{a_1^2}{a_2^2} - 1 \right)$$

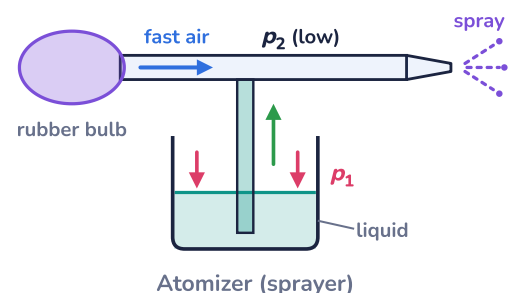


Venturimeter with a manometer

• ATOMIZER OR SPRAYER

The **atomizer** is based on Bernoulli's principle. Pressing the bulb (the forward stroke of the piston) sends a **fast stream of air** past the top of the tube.

So the pressure there (p_2) is **low**. Atmospheric pressure (p_1) on the liquid pushes it up the tube. The liquid meets the fast air and breaks into a **fine spray**.

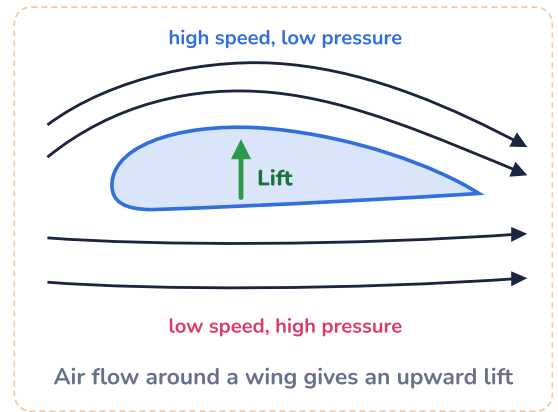


● AEROFOIL (AIRCRAFT WING) LIFT

The wing of an aircraft has a special shape, called an **aerofoil**. Air flows **faster over its top** than along its bottom.

By Bernoulli's principle, the **pressure above** the wing is **lower** than below it. This pressure difference pushes the wing up.

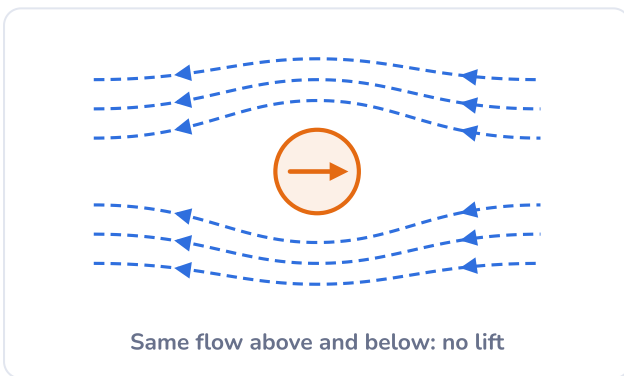
The upward force is called **dynamic lift**.



● SPINNING AND NON-SPINNING BALL

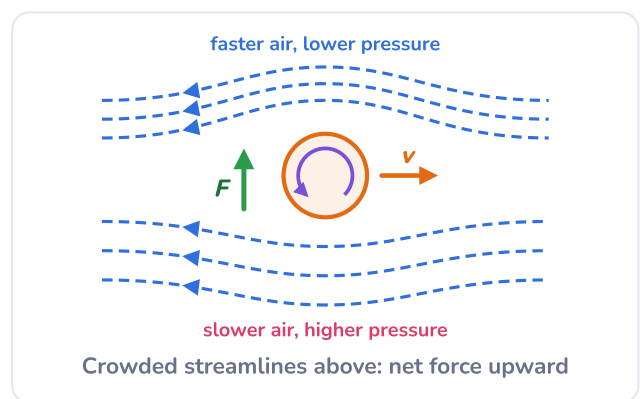
Ball Without Spin

The air moves past the ball with the **same speed above and below** it. So there is **no pressure difference**. The ball feels no force up or down.



Ball With Spin

A spinning ball **drags air** with it. So the air speed is **higher on one side** of the ball. The resulting pressure difference bends its path into a curve. This is called the **Magnus effect**.



3.5 Viscosity

DEFINITION

Let one layer of a liquid move over the next layer. A **tangential friction force** then acts between them. This **backward dragging force** opposes their relative motion.

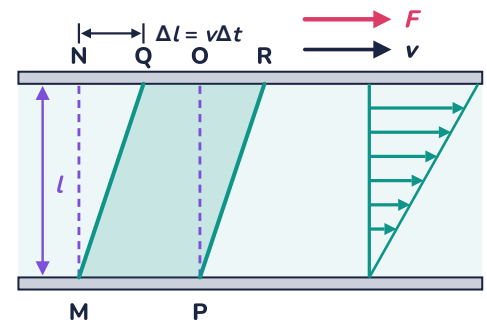
Viscosity is the property of a fluid that opposes the relative motion between its layers. It is also called **fluid friction**.

e.g. **Machine parts** get jammed in winter, because the lubricating oil becomes **more viscous** (thicker) in the cold.

• COEFFICIENT OF VISCOSITY

Take a layer of fluid that is being deformed by a **shearing stress**. The **coefficient of viscosity** η is the **shearing stress** divided by the **rate of change of strain**.

$$\eta = \frac{\text{shearing stress}}{\text{rate of change of strain}}$$



A fluid layer deformed by a shearing force

COEFFICIENT OF VISCOSITY

$$\eta = \frac{F/A}{v/l} = \frac{Fl}{vA} = \frac{F}{A} \frac{dv}{dx}$$

where, F = tangential force, A = area of the layer, $\frac{dv}{dx}$ = velocity gradient. Dimensions: $[ML^{-1}T^{-1}]$

CGS Unit: Poise

$$1 \text{ poise} = 1 \text{ dyne s cm}^{-2}$$

$\eta = 1$ **poise** when a tangential force of 1 dyne per cm^2 of area keeps a relative velocity of 1 cm s^{-1} between two layers 1 cm apart.

SI Unit: Decapoise

$$1 \text{ decapoise} = 10 \text{ poise} = 1 \text{ N s m}^{-2}$$

The SI unit N s m^{-2} (or Pa s) is also called the **decapoise**.

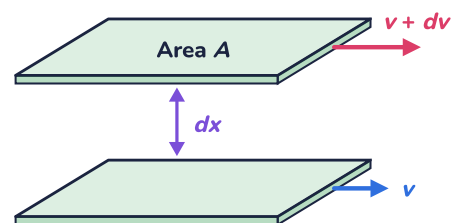
• FLUID FRICTION FORCE (VISCOUS FORCE)

Between two fluid layers, the viscous force **varies directly** as the **area** A of the layers. It also varies directly as the **velocity gradient** $\frac{dv}{dx}$ between them.

VISCOUS (FLUID FRICTION) FORCE

$$F = \eta A \frac{dv}{dx}$$

where, η = coefficient of viscosity, A = area of the layers, $\frac{dv}{dx}$ = velocity gradient. In vector form a minus sign is added. It shows that the force opposes the motion.



Velocity gradient $\frac{dv}{dx}$ between two fluid layers

3.6 Critical Velocity

DEFINITION

Critical velocity is the **limiting speed** of flow of a liquid. Up to this speed, the flow stays **streamline**. Above it, the flow becomes **turbulent**.

The critical velocity v_c of a liquid flowing in a tube depends on:

i Viscosity

Coefficient of **viscosity** of the liquid (η)

ii Density

Density of the liquid (ρ)

iii Radius

Radius of the tube (r)

Write $v_c = k \eta^a \rho^b r^c$, where k is a dimensionless constant. Comparing dimensions on both sides gives $a = 1$, $b = -1$ and $c = -1$. So:

CRITICAL VELOCITY

$$v_c = \frac{k\eta}{\rho r}$$

For streamline flow, v_c should be large. So η should be large, and ρ and r should be small.

Tends to be Streamline

Liquids with **high viscosity** and **low density**, in **narrow pipes**.

Tends to be Turbulent

Liquids with **low viscosity** and **high density**, in **broad pipes**.

3.7 Reynolds Number

DEFINITION

The **Reynolds number** is a **dimensionless number** whose value tells whether the flow of a liquid in a pipe is **steady (streamline) or turbulent**. It was given by **Osborne Reynolds (1842–1912)**.

REYNOLDS NUMBER

$$R_e = \frac{\rho v D}{\eta}$$

where, ρ = density of the liquid, v = its velocity, η = its coefficient of viscosity, D = diameter of the pipe

$$R_e < 1000$$

The flow is **streamline (laminar)**.

$$1000 \text{ to } 2000$$

The flow is **unsteady**. It may switch between laminar and turbulent.

$$R_e > 2000$$

The flow is **turbulent**.

NOTE

The Reynolds number at which turbulence just starts is called the **critical Reynolds number**. The Reynolds number compares two forces, each taken per unit area. It is the **inertial force** divided by the **viscous force**.

$$R_e = \frac{\text{Inertial force}}{\text{Force of viscosity}}$$

3.8 Stokes' Law

CONCEPT

Let a **small sphere** fall through a **viscous fluid at rest**. The fluid layers touching the sphere move along with it. The layers far away stay at rest. This relative motion produces a **backward dragging (viscous) force** that **opposes the motion** of the sphere.

LAW

Stokes' law: A small sphere of radius r moves with velocity v through a fluid of viscosity η . The backward dragging force on it is $F = 6\pi\eta rv$. This law was given by **Sir George G. Stokes (1819–1903)**.

STOKES' LAW (VISCOUS DRAG FORCE)

$$F = 6\pi\eta rv$$

Stokes worked out by calculation that the constant is $k = 6\pi$.

So the viscous drag depends on three things: (i) the coefficient of **viscosity** (η) of the fluid, (ii) the **velocity** (v) of the body and (iii) the **radius** (r) of the spherical body.

GOOD TO KNOW

- ◆ **Examples:** a falling raindrop, a swinging pendulum bob.
- ◆ **Applications:** finding the charge of an electron (Millikan's oil-drop experiment), how clouds form and separate, and how a parachute works.

3.9 Terminal Velocity

DEFINITION

The **terminal velocity** is the **maximum constant velocity** that a body gains while falling through a **viscous fluid**.

A **raindrop** first speeds up as it falls. Its **viscous force grows** with speed. Soon its **weight = upthrust + viscous force**. From then on it falls at a **constant (terminal) velocity**.

SETUP – GIVEN INFORMATION

- ◆ Let a sphere (radius r , density ρ) fall through a fluid of density σ and viscosity η . Its terminal velocity is v .
- ◆ **Weight** of the sphere (downward) $= \frac{4}{3}\pi r^3 \rho g$
- ◆ **Upthrust** = weight of fluid displaced (upward) $= \frac{4}{3}\pi r^3 \sigma g$
- ◆ **Viscous force** (upward, by Stokes' law) $= 6\pi\eta r v$

DERIVATION

- At terminal velocity, weight = upthrust + viscous force:

$$\frac{4}{3}\pi r^3 \rho g = \frac{4}{3}\pi r^3 \sigma g + 6\pi\eta r v$$

- Keep the viscous term on one side:

$$6\pi\eta r v = \frac{4}{3}\pi r^3 (\rho - \sigma) g$$

TERMINAL VELOCITY

$$v = \frac{2}{9} \cdot \frac{r^2 (\rho - \sigma) g}{\eta}$$

where, r = radius of the body, ρ = its density, σ = density of the fluid, η = coefficient of viscosity

3.10 Poiseuille's Formula

CONCEPT

Poiseuille's formula gives the **volume of liquid** that flows each second through a narrow **horizontal tube (capillary tube)**. The tube has length L and radius r . A **pressure difference** p is applied across its ends.

POISEUILLE'S FORMULA

$$V = \frac{Q}{t} = \frac{\pi p r^4}{8\eta L}$$

where, p = pressure difference across the ends, r = radius of the tube, L = length of the tube, η = coefficient of viscosity

Rate of flow V and **radius of the tube** ($V \propto r^4$)

Depends very strongly

Rate of flow V and **length** ($V \propto \frac{1}{L}$) and **viscosity** ($V \propto \frac{1}{\eta}$)

Inversely