

CHAPTER

08

Mechanical Properties of Solids

Stress, Strain and Hooke's Law • Moduli of Elasticity • Elastic Effects and Applications

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What's inside

1 Stress, Strain and Hooke's Law	2	2 Moduli of Elasticity	10
1.1 Elastic Behaviour of Solids	2	2.1 Modulus of Elasticity	10
1.2 Spring Ball Model of Solids	3	2.2 Young's Modulus	11
1.3 Stress	3	2.3 Determination of Young's Modulus	12
1.4 Types of Stress	4	2.4 Bulk Modulus	13
1.5 Strain	5	2.5 Modulus of Rigidity	14
1.6 Hooke's Law	6	2.6 Comparison of the Elastic Moduli	15
1.7 Stress-Strain Curve	7	2.7 Poisson's Ratio	15
1.8 Ductile and Brittle Materials	8	3 Elastic Effects and Applications	16
1.9 Elastomers	9	3.1 Factors Affecting Elasticity	16
1.10 Malleability	9	3.2 Elastic After Effect	17
		3.3 Elastic Fatigue	17
		3.4 Energy Stored in a Deformed Body	17
		3.5 Applications of Elastic Behaviour	18

READING KEY

DEFINITION

NOTE

REMEMBER

DERIVATION

STRATEGY

Formulas sit on graph paper • coloured words are the key terms

A solid has a **definite shape and size**. A **force** is needed to change (deform) its shape or size. This chapter studies how solids react to such forces. It uses the ideas of **stress**, **strain**, **Hooke's law** and the **moduli of elasticity**.

1.1 Elastic Behaviour of Solids

DEFINITION

Elasticity

Elasticity is the property of a body by which it **gets back its original shape and size** after the deforming force is removed. A deformation that disappears on removing the force is called an **elastic deformation**.

DEFINITION

Deforming Force

A **deforming force** is a force that changes the **shape** of a body. It does not change the body's state of rest or of uniform motion.

Even a solid body changes its shape when a suitable force acts on it. You can see this when you **stretch a rubber band** or **hang a load on a spring**. A body can be deformed in **four ways**:

Stretched gets longer

Compressed gets shorter

Bent becomes curved

Twisted turns about its axis

Interatomic (or **intermolecular**) **forces** hold the atoms of a solid together. These forces keep every atom in a **stable equilibrium** position. A solid reacts to a deforming force in two stages:

i When the Force Acts

The atoms are pushed or pulled away from their **equilibrium positions**. So the **distances between the atoms** change.

ii When the Force Is Removed

The interatomic forces **bring the atoms back** to their equilibrium positions. This is the cause of **elastic behaviour**.

Force **applied**: the gaps between atoms change

Atoms pushed out of place

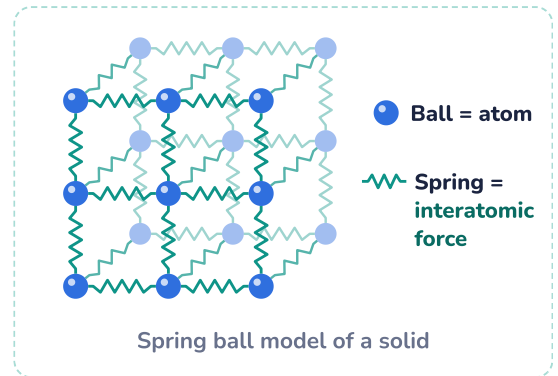
Force **removed**: restoring forces pull the atoms

Atoms go back

1.2 Spring Ball Model of Solids

In the **spring ball model**, the atoms of a solid are treated as **small balls** (point masses). Springs join these balls in all **three dimensions**. The springs stand for the **interatomic forces of attraction**.

Normally, each ball rests where its **potential energy is minimum**. Here the interatomic force on it is **zero**. If a ball is moved away from this mean position, the springs attached to it pull it back. This pull is the **restoring force**.



A ball is moved away from its **mean position**

Springs pull it back

NOTE

Microscopic View

This model explains elastic behaviour through the **microscopic** structure of a solid. Whenever atoms leave their equilibrium positions, the "springs" between them produce **restoring forces**.

1.3 Stress

A deforming force shifts the atoms of a body from their normal positions. This changes its **configuration** (shape or size). An internal **restoring force** then tries to bring the body back.

DEFINITION

Stress is the **restoring force** set up inside a deformed body, **per unit area**.

STRESS

$$\text{Stress} = \frac{F}{A}$$

where, F = restoring force (N) and A = area of cross-section (m^2). If the body is not permanently deformed, the restoring force is **equal and opposite** to the applied force. So F can be taken as the **external deforming force**.

• UNITS AND DIMENSIONS OF STRESS

SI Unit

$\text{N/m}^2 = \text{Pa}$ (newton per square metre, or pascal)

CGS Unit

dyne/cm^2 (dyne per square centimetre)

Dimensions

$[\text{ML}^{-1}\text{T}^{-2}]$, the same as for pressure

Stress cannot be given one fixed direction

Not a vector quantity

Breaking stress of a given material

Constant

1.4 Types of Stress

Stress is of **three types**. The type depends on how the forces act on the body.

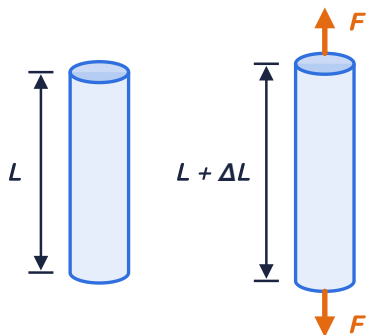
1. NORMAL (LONGITUDINAL) STRESS

Normal stress is the restoring force per unit area that acts **perpendicular** (normal) to the surface of a body. Normal stress is either **tensile** or **compressive**.

i Tensile Stress

Equal and opposite forces **pull** the two ends of a rod, normal to its cross-section. The rod gets **longer**. A restoring force equal to F develops inside. This force per unit area is the **tensile stress**.

$$\text{Tensile stress} = \frac{F}{A}$$

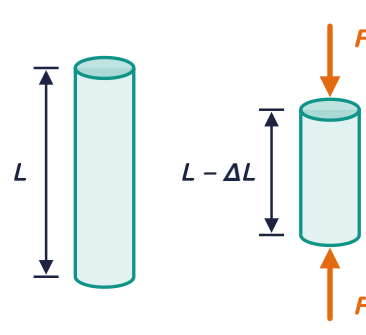


Pulled by F at both ends: length $L \rightarrow L + \Delta L$

ii Compressive Stress

Equal and opposite forces **push** the two ends of a rod. The rod gets **shorter**. Again, a restoring force equal to F develops inside. This force per unit area is the **compressive stress**.

$$\text{Compressive stress} = \frac{F}{A}$$



Pushed by F at both ends: length $L \rightarrow L - \Delta L$

NOTE

Tensile and compressive stress together are called **longitudinal stress**. In both cases the **net force** on the body is **zero**. Even so, the body gets **deformed**.

2. TANGENTIAL (SHEARING) STRESS

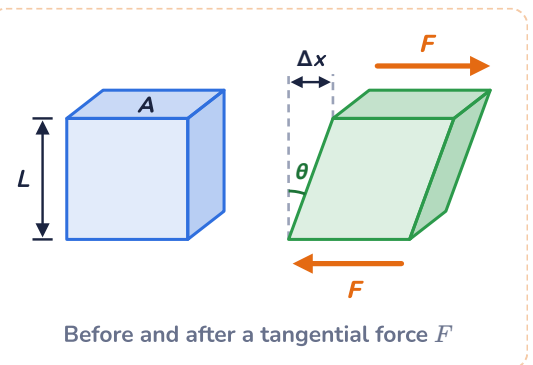
A deforming force may act **along** a surface (**tangentially**). Then the **shape** changes, but the **volume stays the same**. This force per unit area is the **tangential** or **shearing stress**.

In the figure, F acts along the top face of a cube. The top face slides by Δx . The sides tilt through the **angle of shear** θ .

$$\text{Tangential stress} = \frac{F}{A}$$

Force acts **along the top face**

Shape changes, volume same



Before and after a tangential force F

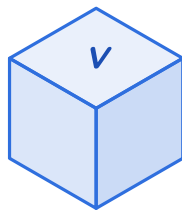
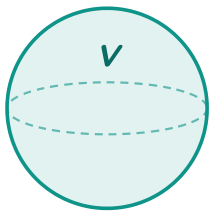
• 3. HYDRAULIC (BULK) STRESS

Sometimes a body is pressed by a **uniform force from all sides**. The stress produced is called **hydraulic stress** or **bulk stress**. The **volume** of the body changes, but its **shape** does not.

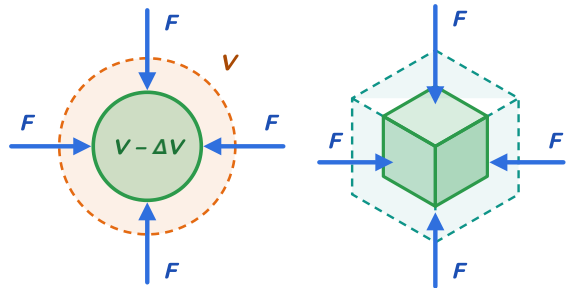
The force F acts **normal to every point** of the surface. It changes the volume of the body by ΔV .

$$\text{Bulk stress} = \frac{F}{A}$$

Hydraulic stress is also called **volume stress**. In general, stress causes a **distortion**: a change in the size, the shape, or both (the configuration) of the body.



Before: original volume V



After: forces from all sides, volume falls by ΔV

The same force acts normally at every point of the surface

Volume changes, shape same

1.5 Strain

A deforming force changes the shape or size of a body. The body is then said to be **strained** or **deformed**.

DEFINITION

Strain is the ratio of the **change in configuration** of a body to its **original configuration**.

STRAIN

$$\text{Strain} = \frac{\text{Change in configuration}}{\text{Original configuration}}$$

Strain is a ratio of **two similar quantities**. So it has **no unit** and **no dimensions**.

• THREE TYPES OF STRAIN (LENGTH, VOLUME, SHAPE)

1 Longitudinal Strain

It is the **change in length** ÷ original length.

$$\text{Longitudinal strain} = \frac{\Delta L}{L}$$

2 Volumetric Strain

It is the **change in volume** ÷ original volume.

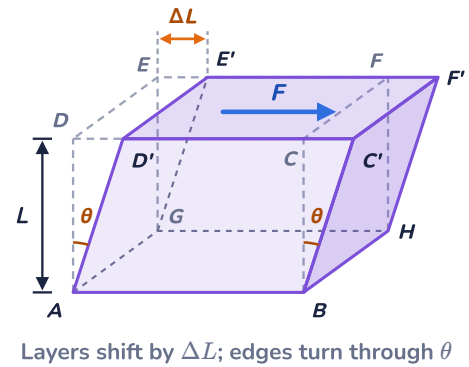
$$\text{Volumetric strain} = \frac{\Delta V}{V}$$

3 Shear Strain

A strain that changes the **shape** of a body is called **shear strain**.

It equals the **relative displacement** of a layer divided by its **distance from the fixed layer**. It is also measured by the **angle of shear** θ . It is the angle by which a line, first perpendicular to the fixed face, turns.

$$\text{Shear strain} = \tan \theta = \frac{\Delta L}{L}$$



If $L = 1 \text{ m}$, then shear strain $= \Delta L$

Relative shift of layers 1 m apart

NOTE

- ◆ When a load hangs from a spring, the spring gets both **longitudinal** and **shear** strain.
- ◆ Strain can **remain for some time** even after the stress is removed. So strain **lags behind** stress.

1.6 Hooke's Law

Robert Hooke (1635–1703) was an English physicist. Based on his experiments, he gave a law in **1679**. It is now called **Hooke's law**.

LAW

Original Statement

Within the **elastic limit**, the **extension** of a wire is **directly proportional** to the **load** hung on it.

ORIGINAL FORM

$$\text{Extension} \propto \text{Load applied}$$

MODIFIED (GENERAL) FORM

Later, the law was found to hold for **every kind of deformation**:

Stretching

Compression

Bending

Twisting

LAW

Hooke's Law (Modified Statement)

Within the **elastic limit**, **stress** is **directly proportional** to **strain**.

$$\text{Stress} \propto \text{Strain} \Rightarrow \text{Stress} = E \times \text{Strain}$$

$$E = \frac{\text{Stress}}{\text{Strain}}$$

where, E = a constant called the **modulus of elasticity** of the material

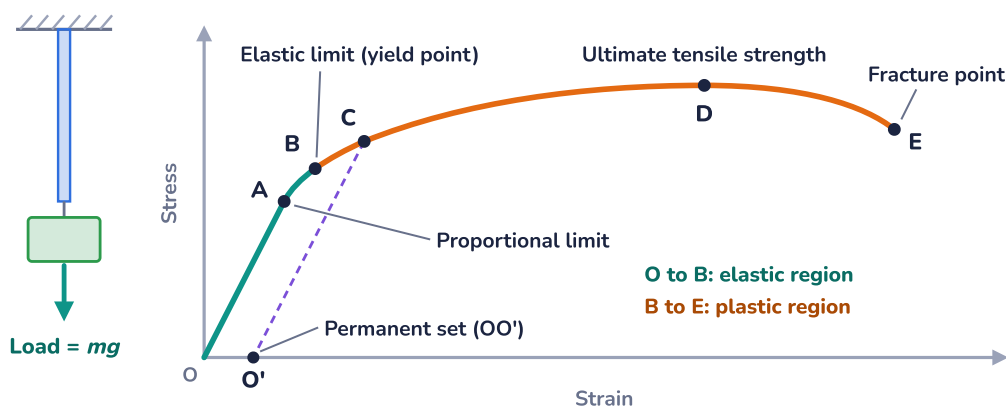
NOTE

The constant E depends only on the **nature of the material**. It has three forms: **Young's modulus**, **bulk modulus** and **modulus of rigidity**. Part 2 explains each of them.

1.7 Stress-Strain Curve

Hang a load on a wire and increase the load slowly. For **small loads**, the extension is proportional to the load. So **stress $\propto$ strain**.

The graph of **stress** against **strain** is called the **stress-strain curve**. With a growing load, the wire first behaves as an **elastic** body and then as a **plastic** body. In the end, it breaks.



DEFINITION

Elastic Limit

The **elastic limit** is the **maximum deforming force** (or stress) up to which a body regains its original form completely once the force is removed. Beyond this limit, the body loses its elasticity and becomes **permanently deformed**.

The elastic limit is a property of a **body**, but elasticity is a property of its **material**.

• REGIONS OF THE STRESS-STRAIN CURVE

1 O to A: Proportional Region

The graph is a **straight line**. Stress is proportional to strain, so **Hooke's law is fully obeyed**. Point A is the **proportional limit**.

2 O to B: Elastic Region

Remove the load anywhere before B, and the wire **returns to its original length**. B is the **elastic limit**, also called the **yield point**.

3 B to C: Permanent Set

Let the stress go beyond B. Then remove the load, say at C. The wire **does not** regain its original length. A **permanent strain** OO' remains. It is called the **permanent set**.

4 C to E: Plastic Region and Fracture

Now a small increase in stress gives a **large strain**. This is **plastic behaviour**. The stress is highest at D, the **ultimate tensile strength**. Finally the wire breaks at E, the **fracture point**.

IMPORTANT

Breaking Stress (Ultimate Tensile Strength)

The **breaking stress** of a material is the **largest stress** it can bear before it **breaks**. It is also called the **ultimate tensile strength** (point D on the curve).

● BREAKING STRESS OF SOME MATERIALS (N/M²)

Aluminium

2.2×10^8

Iron

3.0×10^8

Brass

4.7×10^8

Phosphor bronze

5.5×10^8

Steel

$(5 \text{ to } 20) \times 10^8$

Glass

10×10^8

NOTE

- ◆ Hooke's law holds only in the **straight-line part** of the curve. It does **not hold for large strains**.
- ◆ The elastic limit lies **very close** to the proportional limit. So Hooke's law is nearly true right up to the elastic limit.
- ◆ The strain in the **yield region** is **15 to 20 times** the strain in the proportional region.

1.8 Ductile and Brittle Materials

Materials can be grouped by their **elastic** and **plastic** behaviour. This topic and the next two describe these groups.

i Ductile Materials

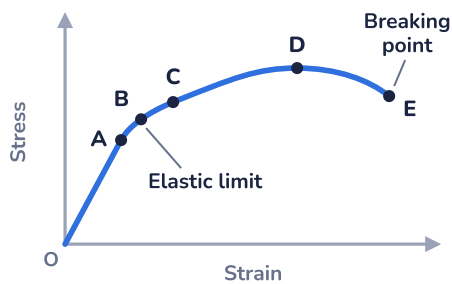
Ductile materials have a **large plastic range** of extension. Their fracture point is **far from** the elastic limit. So they get much longer, permanently, before they break. Thin wires can be **drawn** from them.

e.g. Iron, copper, aluminium, silver

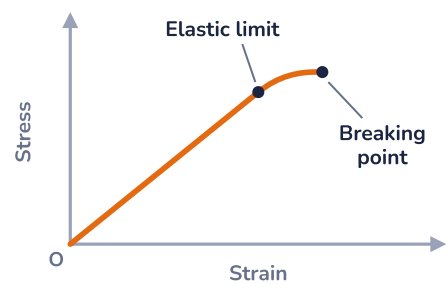
ii Brittle Materials

Brittle materials have a **very small plastic range**. They break as soon as the stress goes a little beyond the **elastic limit**. Their breaking point lies **very close** to the elastic limit.

e.g. Glass, cast iron, ceramics



Ductile: long plastic range



Brittle: breaks just after the elastic limit

1.9 Elastomers

DEFINITION

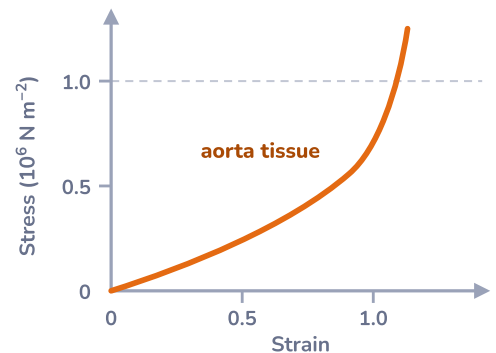
Elastomers are materials that can be stretched **elastically** to very **large strains**.

e.g. **Rubber** can become several times longer than it was. Yet it gets back its first length once the force is removed.

Elastomers have **no clear plastic region**. Their elastic region is very large, but they **do not obey Hooke's law**. The graph has no straight-line part.

Very large elastic strain (rubber, aorta)

Does not obey Hooke's law



Stress-strain curve of an elastomer (elastic tissue of the aorta)

NOTE

The graph above is for the elastic tissue of the **aorta**, which is an elastomer. The aorta is the large blood vessel that carries blood from the heart.

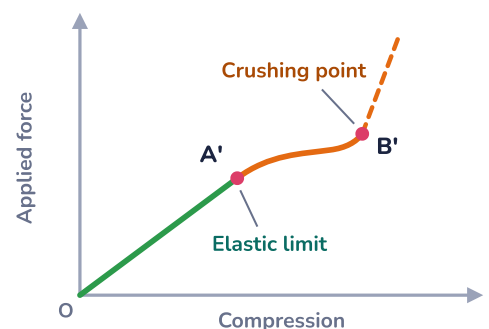
1.10 Malleability

A **compressed** solid reaches a stage beyond which it cannot regain its shape. This stage is its **elastic limit for compression**. Beyond it, the solid acts like a **plastic body**.

More compression brings a metal to its **crushing point**. Beyond this point, some metals can be **flattened into thin sheets**.

DEFINITION

A metal that can be flattened into thin sheets by **hammering or rolling** is called **malleable**. This property is called **malleability**. **e.g.** Silver, lead.



Load-compression curve for a metal

Within the elastic limit, stress divided by strain is a **constant** for a material. This constant is its **modulus of elasticity**. This part covers its three types and **Poisson's ratio**.

2.1 Modulus of Elasticity

DEFINITION

The **modulus of elasticity** of a body is the **ratio of stress to strain** within the elastic limit. Another name for it is the **coefficient of elasticity**.

MODULUS OF ELASTICITY

$$E = \frac{\text{Stress}}{\text{Strain}}$$

• UNITS AND PROPERTIES

SI Unit

$\text{Nm}^{-2} = \text{Pa}$ (newton per square metre, or pascal)

Dimensions

$[\text{ML}^{-1}\text{T}^{-2}]$, the same as for stress or pressure

Changes with the **material** of the body

Yes

Changes with the **size** of the body (length, area, volume, etc.)

No

• THREE TYPES OF MODULUS OF ELASTICITY

1 Young's Modulus (Y)

For a change in **length**:
longitudinal stress ÷
longitudinal strain.

$$Y = \frac{\text{Long. stress}}{\text{Long. strain}}$$

2 Bulk Modulus (B or K)

For a change in **volume**: normal
(volume) stress ÷ volumetric
strain.

$$B = \frac{\text{Normal stress}}{\text{Vol. strain}}$$

3 Modulus of Rigidity (η)

For a change in **shape**:
tangential (shear) stress ÷
shear strain.

$$\eta = \frac{\text{Shear stress}}{\text{Shear strain}}$$

2.2 Young's Modulus

DEFINITION

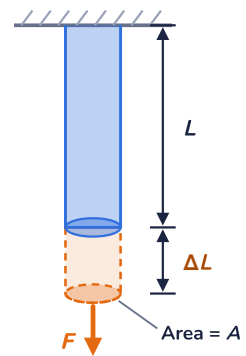
The **Young's modulus** (Y) of a material is the ratio of **longitudinal stress** to **longitudinal strain**, within the elastic limit.

YOUNG'S MODULUS

$$Y = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}}$$

SETUP – GIVEN INFORMATION

- ◆ A wire has radius r and length L .
- ◆ A force F acts along its length, that is, normal to its cross-section.
- ◆ The length of the wire increases by ΔL .
- ◆ Area of the circular cross-section, $A = \pi r^2$.



Wire stretched by a force F

DERIVATION

- 1 For the stretched wire:

$$\text{Longitudinal stress} = \frac{F}{A}, \quad \text{Longitudinal strain} = \frac{\Delta L}{L}$$

- 2 Young's modulus is the ratio of these two:

$$Y = \frac{F/A}{\Delta L/L} = \frac{FL}{A \Delta L} \quad \dots (i)$$

- 3 For a circular cross-section, $A = \pi r^2$. Put this in (i):

$$Y = \frac{FL}{\pi r^2 \Delta L}$$

- 4 If a load of mass m stretches the wire, then $F = mg$:

$$Y = \frac{mgL}{\pi r^2 \Delta L}$$

$$Y = \frac{FL}{\pi r^2 \Delta L} = \frac{mgL}{\pi r^2 \Delta L}$$

where, $F = mg$ = stretching force, L = original length, r = radius and ΔL = extension

CONCEPT

Physical Meaning of Young's Modulus

Put $L = 1$ m, $A = 1$ m² and $\Delta L = 1$ m. Then $Y = F$. So Young's modulus equals the **force needed to stretch** a wire of **unit length** (1 m) and **unit cross-section** (1 m²) by **unit amount** (1 m).

$$\Delta L \propto F \text{ and } \Delta L \propto L$$

More force or a longer wire gives more extension

2.3 Determination of Young's Modulus

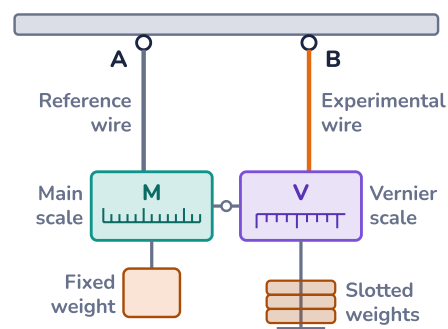
The simple set-up below is used to find the **Young's modulus** of the material of a wire. Two long, straight wires of the **same length** and **same radius** hang side by side from a **rigid support**.

Reference Wire (A)

It carries the **main scale M** (a millimetre scale). A **heavy fixed weight** keeps it **taut** and free of kinks.

Experimental Wire (B)

It carries a **vernier scale V**, which slides along the main scale. Its **hanger** holds **slotted half-kg weights**.



Experimental set-up (Searle's apparatus)

PROCEDURE

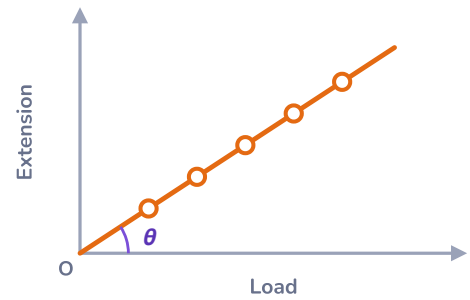
- 1 Measure the radius of the experimental wire with a **screw gauge** at several places. Take the average as the radius r . Note the initial length L .
- 2 Put a small **initial load** (about 1 kg) on the hanger. It keeps the wire straight and free of kinks. Note the **vernier reading**.
- 3 Add a **half-kg weight**. Let the wire stretch for about **one minute**. Note the new vernier reading.
- 4 The **difference** between the two readings gives the extension ΔL for the added weight. Repeat with more weights, one step at a time, and note each extension.

The cross-section of wire B is $A = \pi r^2$. The stretching force is $F = mg$.

A graph is drawn between the **load** and the **extension**. It is a **straight line through the origin**. This shows that the extension is **directly proportional** to the load.

YOUNG'S MODULUS FROM THE EXPERIMENT

$$Y = \frac{FL}{\pi r^2 \Delta L} = \frac{mgL}{\pi r^2 \Delta L}$$



A straight line through the origin O

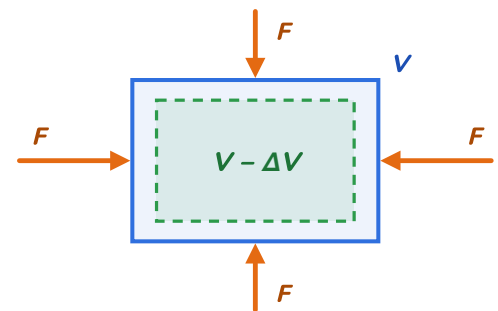
2.4 Bulk Modulus

DEFINITION

The **bulk modulus** (B) of a material is the ratio of **normal stress** to **volumetric strain**, within the elastic limit. It also equals **hydraulic stress** ÷ **hydraulic strain**.

Take a body of volume V and surface area A . A force F acts **evenly over its whole surface**, normal to it. The volume of the body decreases by ΔV .

$$B = \frac{\text{Normal stress}}{\text{Volumetric strain}} = \frac{F/A}{\Delta V/V}$$



Volume V becomes $V - \Delta V$

BULK MODULUS

$$B = -\frac{pV}{\Delta V}$$

where, $p = \frac{F}{A}$ = normal pressure. The minus sign shows that the volume **decreases** when the stress increases.

NOTE

- ◆ The **negative sign** shows that the volume falls as the stress rises. So, for a system in equilibrium, B is always **positive**.
- ◆ A **perfectly rigid body** and an **ideal liquid** both have an **infinite** bulk modulus.

● COMPRESSIBILITY

DEFINITION

The **compressibility** (k) of a material is the **reciprocal of its bulk modulus**.

COMPRESSIBILITY

$$k = \frac{1}{B} = -\frac{\Delta V}{pV}$$

SI Unit

$$\text{N}^{-1}\text{m}^2$$

CGS Unit

$$\text{dyne}^{-1}\text{cm}^2$$

Dimensions

$$[\text{M}^{-1}\text{LT}^2]$$

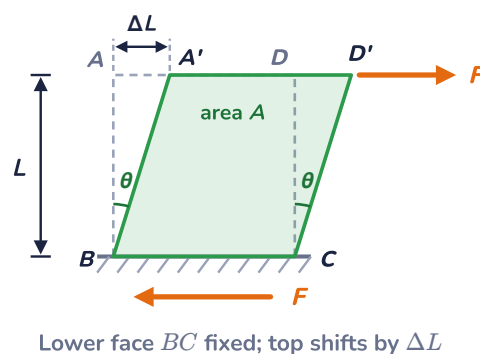
2.5 Modulus of Rigidity

DEFINITION

The **modulus of rigidity** (or **shear modulus**) of a material is the ratio of **tangential (shear) stress** to **shear strain**, within the elastic limit. It is written as G or η .

Take a cube with its **lower face fixed**. A tangential force F acts on its **upper face** of area A .

The upper face shifts by $AA' = \Delta L$. The vertical sides turn through an angle θ . This angle is the **shear strain**.



MODULUS OF RIGIDITY

$$\eta \text{ (or } G) = \frac{\text{Tangential (shear) stress}}{\text{Shear strain}}$$

DERIVATION

- The tangential (shear) stress on the upper face is:

$$\text{Shear stress} = \frac{F}{A}$$

- In the figure, $AB = DC = L$ and $\angle ABA' = \theta$. For a small angle, the shear strain is:

$$\theta \approx \tan \theta = \frac{AA'}{AB} = \frac{\Delta L}{L}$$

- So the modulus of rigidity is:

$$\eta = \frac{F/A}{\theta} = \frac{F}{A\theta} = \frac{F}{A} \cdot \frac{L}{\Delta L}$$

SHEAR MODULUS

$$\eta = \frac{FL}{A \Delta L}$$

2.6 Comparison of the Elastic Moduli

i Solids

A solid has **all three** moduli of elasticity: Young's modulus Y , bulk modulus B and modulus of rigidity η .

ii Fluids (Liquids and Gases)

A fluid can change only its **volume**, not its shape. So a fluid has **only the bulk modulus** B . For an **ideal liquid**, the modulus of rigidity is **zero**.

• SUMMARY: STRESS, STRAIN AND THE ELASTIC MODULI

TYPE OF STRESS	NATURE OF FORCE	STRAIN	CHANGE IN SHAPE	CHANGE IN VOLUME	ELASTIC MODULUS	STATE OF MATTER
Tensile / Compressive	Two equal and opposite forces $\perp$ to opposite faces	Longitudinal, $\frac{\Delta L}{L}$	Yes	No	Young's modulus $Y = \frac{FL}{A \Delta L}$	Solid
Shearing	Two equal and opposite forces $\parallel$ to opposite faces (net force and torque zero)	Pure shear, θ	Yes	No	Shear modulus $\eta = \frac{F}{A \theta}$	Solid
Hydraulic (Bulk)	Forces $\perp$ everywhere on the surface; pressure the same all over	Volumetric, $\frac{\Delta V}{V}$	No	Yes	Bulk modulus $B = -\frac{pV}{\Delta V}$	Solid, liquid and gas

2.7 Poisson's Ratio

A loaded wire becomes longer and thinner. Its **length increases**, and its **diameter decreases**. The strain along the force is the **longitudinal strain**. The strain at right angles to the force is the **lateral strain**.

DEFINITION

The ratio of **lateral strain** to **longitudinal strain** is called **Poisson's ratio** (σ).

i Longitudinal Strain

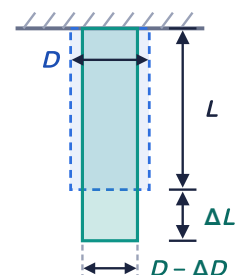
The length increases from L to $L + \Delta L$.

$$\frac{\Delta L}{L}$$

ii Lateral Strain

The diameter decreases from D to $D - \Delta D$.

$$-\frac{\Delta D}{D}$$



POISSON'S RATIO

$$\sigma = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} = \frac{-\Delta D/D}{\Delta L/L} = -\frac{L \Delta D}{D \Delta L}$$

The **minus sign** shows that the two strains are in **opposite senses**. One is an increase, and the other is a decrease. σ is a ratio of two strains, so it has **no unit** and **no dimensions**. It is a **constant** for a material, as long as the material stays **elastic, homogeneous and isotropic**.

For **theoretical** purposes

$$-1 < \sigma < 0.5$$

For **practical** purposes

$$0 < \sigma < 0.5$$

RELATIONS BETWEEN THE ELASTIC CONSTANTS

$$(i) Y = 3B(1 - 2\sigma)$$

$$(ii) Y = 2\eta(1 + \sigma)$$

$$(iii) \sigma = \frac{3B - 2\eta}{2\eta + 6B}$$

$$(iv) \frac{9}{Y} = \frac{1}{B} + \frac{3}{\eta}$$

where, Y = Young's modulus, B = bulk modulus, η = modulus of rigidity and σ = Poisson's ratio

03

PART 3

Elastic Effects and Applications

What changes **elasticity**, the **energy stored** in a wire, and everyday **uses of elasticity**.

3.1 Factors Affecting Elasticity

i Hammering and Rolling

Elasticity ↑

Both processes **break the crystal grains** into smaller pieces. This **raises** the elasticity.

ii Annealing

Elasticity ↓

Annealing makes the **crystal grains bigger**. This **lowers** the elasticity.

iii Presence of Impurities

↑ or ↓

An impurity can **increase** or **decrease** the elasticity. It depends on the **nature of the impurity**.

iv Temperature

Elasticity ↓

For most materials, elasticity **decreases** as the temperature rises. **Exception:** special nickel-steel alloys such as **elinvar** (a kind of invar steel). Their elasticity stays almost the same when the temperature changes.

3.2 Elastic After Effect

DEFINITION

The **delay** of a body in getting back its original state, after the deforming force is removed, is called the **elastic after effect**.

When the deforming force is removed, some bodies return to their original state **at once**. Others take a **long time**.

i Quartz Fibre

Negligible effect

It returns to its normal state **immediately** when the twisting torque on it stops.

ii Glass Fibre

Large effect

It needs **hours** to recover its original state once the force is removed.

3.3 Elastic Fatigue

DEFINITION

Elastic fatigue is the **loss of strength** of a material when it goes through **repeated alternating strains**.

i Metal Wire

Bend a metal wire **again and again** in opposite directions. It slowly loses strength due to elastic fatigue. Finally it **breaks**.

ii Railway Bridges

Railway bridges are declared **unsafe** after a long period of use. This prevents accidents due to elastic fatigue.

3.4 Energy Stored in a Deformed Body

CONCEPT

Elastic Potential Energy

Stretching a wire brings **interatomic forces** into play. They oppose the change. So **work** must be done against these restoring forces. This work is stored in the wire as **elastic potential energy**.

SETUP – GIVEN INFORMATION

- ◆ A force F on a wire of length L increases its length by ΔL .
- ◆ At the start, the wire has **no** internal restoring force.
- ◆ As the length grows by ΔL , the internal force rises from 0 to F .

DERIVATION

- 1 The average internal force during the stretching is:

$$F_{avg} = \frac{0 + F}{2} = \frac{F}{2}$$

- 2 Work done = average force $\times$ increase in length. It is stored as elastic potential energy U :

$$U = \frac{F}{2} \times \Delta L = \frac{1}{2} F \Delta L$$

- 3 Let A be the area of cross-section. Multiply and divide by AL :

$$U = \frac{1}{2} \cdot \frac{F}{A} \cdot \frac{\Delta L}{L} \times AL$$

- 4 Here $\frac{F}{A} = \text{Stress}$, $\frac{\Delta L}{L} = \text{Strain}$ and $AL = \text{Volume}$. So:

$$U = \frac{1}{2} \times \text{Stress} \times \text{Strain} \times \text{Volume}$$

ELASTIC POTENTIAL ENERGY STORED

$$U = \frac{1}{2} \times \text{Stress} \times \text{Strain} \times \text{Volume}$$

The energy stored in **unit volume** of the wire is called its **elastic energy density** (μ).

ELASTIC ENERGY DENSITY

$$\mu = \frac{U}{\text{Volume}} = \frac{1}{2} \text{Stress} \times \text{Strain} = \frac{1}{2} Y \times (\text{Strain})^2$$

The last form uses Stress = $Y \times \text{Strain}$.

3.5 Applications of Elastic Behaviour

Elastic behaviour matters in **engineering** and in **daily life**. A structure must carry its load without **permanent deformation** or **breaking**. So its design depends on the **elastic limit** of the material used. Four important uses are given below.

• (I) PARTS OF MACHINERY

A metal part of a machine is **never** loaded beyond the **elastic limit** of its material.

Stress kept **within** the elastic limit

Part works properly ✓

Stress taken **beyond** it: the machine does not work properly

Permanently deformed ✗

• (II) METAL ROPES IN CRANES

Cranes lift and move heavy loads using **metal ropes**. The **thickness** of such a rope is chosen from its **elastic limit** and a suitable **factor of safety**.

NOTE

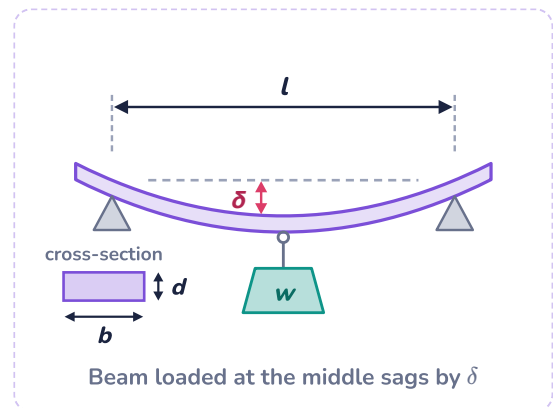
Factor of Safety

The **factor of safety** is also called the **safety factor (SF)**. It tells how much **extra load** a structure can carry beyond its **expected (actual) load**.

• (III) DESIGN OF BRIDGES AND BEAMS

A **bridge** or a **beam** must carry moving traffic, resist the **wind** and also support its **own weight**. So its **bending** under a load is very important. It must not **bend too much**, and it must not **break**.

Take a beam of length l with a **rectangular cross-section** of breadth b and depth d . It rests on two supports. A load w hangs from its **middle**. The beam sags (is depressed) by an amount δ .



DEPRESSION OF A CENTRALLY LOADED BEAM

$$\delta = \frac{w l^3}{4 Y b d^3}$$

where, l = length, b = breadth, d = depth, Y = Young's modulus and w = load at the centre. The sag δ varies **inversely** as b and **inversely** as d^3 .

Sag and breadth: $\delta \propto \frac{1}{b}$

Weak dependence

Sag and depth: $\delta \propto \frac{1}{d^3}$

Strong (cube) dependence

REMEMBER

Key Design Point

Since $\delta \propto 1/d^3$ but only $\delta \propto 1/b$, increasing the **depth** d reduces the sag **much more** than increasing the breadth b .

• BENDING RESISTANCE AND ELASTIC MODULUS

A beam carries a load by **resisting bending**. The greater this resistance, the **more load** the beam can carry. This resistance depends on three things:

Elastic modulus of material

Cross-section shape of beam

Ultimate stress of material

• (IV) MAXIMUM HEIGHT OF A MOUNTAIN

The elastic behaviour of the Earth sets the **greatest possible height of a mountain** on Earth. It is about **10 km**. The base of a mountain is **not under uniform compression**. So the rocks at the base feel a **shearing stress**. Under this stress, the rocks can **flow**.

SETUP – GIVEN INFORMATION

- ◆ Height of the mountain = h . Density of its material, $\rho \approx 3 \times 10^3 \text{ kg/m}^3$.
- ◆ Elastic limit of a typical rock = $30 \times 10^7 \text{ N/m}^2$. Take $g = 9.8 \text{ m/s}^2$.

DERIVATION

- 1 The weight of the mountain presses on its base. The stress there is:

$$\text{Stress at base} = h\rho g$$

- 2 At the greatest height, this stress just equals the **elastic limit** of the rock:

$$h\rho g = 30 \times 10^7$$

- 3 Put $\rho = 3 \times 10^3 \text{ kg/m}^3$ and $g = 9.8 \text{ m/s}^2$:

$$h \times 3 \times 10^3 \times 9.8 = 30 \times 10^7$$

- 4 Solve for h :

$$h = \frac{30 \times 10^7}{2.94 \times 10^4} \approx 1.0 \times 10^4 \text{ m} \approx 10 \text{ km}$$

MAXIMUM HEIGHT OF A MOUNTAIN

$$h_{\text{max}} \approx 10 \text{ km}$$

Above this height, the stress at the base would cross the elastic limit of the rock. The rock would then **flow**. So mountains on Earth cannot be much taller.