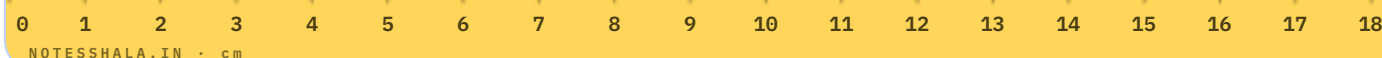


CHAPTER

01

Units and Measurements

System of Units • Significant Figures & Rounding Off • Dimensions of a Physical Quantity



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READING KEY

DEFINITION

NOTE

REMEMBER

DERIVATION

STRATEGY

Formulas sit on graph paper • coloured words are the key terms

DEFINITION

System of Units

A few quantities are chosen first, and each one is given its own unit. These units are called **base** (or **fundamental**) **units**. All other units are made by multiplying or dividing powers of the base units. They are called **derived units**. The complete set of base units and derived units is called a **system of units**.

1.1 Physical Quantities

A quantity that can be **measured** is called a **physical quantity**. It is measured directly with an instrument or found by calculation. The **laws of physics** are written using these quantities. Examples: length, mass, speed and force. Physical quantities are of **two types**:

i Fundamental Quantities

These quantities are **not defined using any other quantity**. They are **independent**. They are also called **base quantities**.

e.g. Mass, length, time, temperature, electric current, luminous intensity and amount of substance.

ii Derived Quantities

These quantities are **made from the fundamental quantities** by multiplying or dividing them.

e.g. Velocity (length ÷ time), acceleration and linear momentum.

1.2 Measurement of Physical Quantities

Measurement means **comparing** a quantity with a fixed standard of the **same kind**. This fixed standard is called the **unit**. So every measurement has **two parts**:

1 Unit (u)

The **standard** that the quantity is compared with. For example, the metre, the second or the kilogram.

2 Numerical Value (n)

The **number** written before the unit. It tells **how many times** the unit (u) is contained in the quantity.

NOTE

Smaller Unit, Bigger Number

For the same quantity, the **numerical value** (n) is **inversely proportional** to the **size of the unit** (u):

$$n \propto \frac{1}{u} \Rightarrow nu = \text{constant}$$

e.g. the same length written in three different units:

in metre (m) → $n = 1$ largest unit
in centimetre (cm) → $n = 100$
in millimetre (mm) → $n = 1000$ smallest unit

KEY A smaller unit ($u \downarrow$) gives a bigger number ($n \uparrow$). The product $n \cdot u$ never changes

Changing the unit does **not** change the quantity itself. Let n_1 and n_2 be the numerical values of the same quantity in units u_1 and u_2 . Then $n_1 u_1 = n_2 u_2$.

PHYSICAL QUANTITY

$$Q = nu$$

Q = physical quantity, n = numerical value, u = unit

1.3 Physical Unit

DEFINITION

A **physical unit** is a fixed standard amount of a quantity. It is used to measure other amounts of the **same kind** of quantity.

A good unit must meet a few **basic requirements**:

What Makes a Good Unit

- ◆ It should be of a **convenient size**. It should not be too large or too small for everyday use.
- ◆ It should be **easily available** to anyone who needs it.
- ◆ It should **not change with time**.
- ◆ It should be **easy to reproduce** anywhere in the world.
- ◆ It should **not change with physical conditions** such as pressure, volume or temperature.

Units are of **two types**:

i Fundamental Units

These units **cannot be obtained from one another**. They cannot be broken down into simpler units. They are the units of fundamental quantities, such as **length, mass and time**. They are also called **base units**.

ii Derived Units

These are the units of **all other quantities**. They are formed by combining fundamental units.

e.g. **Speed** is distance divided by time. So its unit is made from the units of distance and time. It is a **derived unit**:

$$\begin{aligned}\text{Speed} &= \frac{\text{Distance}}{\text{Time}} \\ \therefore \text{Unit of speed} &= \frac{\text{Unit of distance}}{\text{Unit of time}} \\ &= \frac{\text{m}}{\text{s}} = \text{ms}^{-1}\end{aligned}$$

1.4 The International System of Units

A **system of units** gives units for all physical quantities, both fundamental and derived. Different systems have been used over the years. The most common ones in mechanics are:

i FPS System

It is the **British** engineering system. It measures length in **foot**, mass in **pound** and time in **second**.

ii CGS System

It is also called the **Gaussian** system. It measures length in **centimetre**, mass in **gram** and time in **second**.

iii MKS System

It measures length in **metre**, mass in **kilogram** and time in **second**.

iv International System of Units (SI Units)

SI is short for **Système International d'Unités**. This French name means International System of Units. It is the system **accepted all over the world**. The **General Conference on Weights and Measures** adopted it in **1960**. In **1971**, the mole was added as its seventh base unit. It gives one standard set of symbols, units and abbreviations for science, technology, industry and trade.

NOTE

SI was developed from the MKS system. So it is also called the **rationalised MKS system**. It has units for every physical quantity used in physics.

1.5 SI Base Quantities and Units

SI has **seven base units**. It also has **two supplementary units** for measuring angles.

● THE SEVEN BASE UNITS AT A GLANCE

LENGTH m metre	MASS kg kilogram	TIME s second	ELECTRIC CURRENT A ampere	THERMODYNAMIC TEMPERATURE K kelvin	AMOUNT OF SUBSTANCE mol mole	LUMINOUS INTENSITY cd candela
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● FORMAL DEFINITIONS (SINCE 2019)

BASE QUANTITY	NAME	SYMBOL	DEFINITION
Length	Metre	m	Fixed by the speed of light: c is exactly $299\,792\,458\text{ m s}^{-1}$. So one metre is the distance light travels in vacuum in $\frac{1}{299\,792\,458}$ of a second. 2019 Same meaning as the 1983 definition.
Mass	Kilogram	kg	Fixed by the Planck constant: h is exactly $6.626\,070\,15 \times 10^{-34}\text{ J s}$, where $\text{J s} = \text{kg m}^2\text{s}^{-1}$. With the metre and the second already fixed, this sets the kilogram. 2019 Earlier (1889–2019): the mass of a platinum–iridium cylinder kept near Paris.
Time	Second	s	Fixed by the caesium-133 atom: the frequency of its hyperfine transition, $\Delta\nu_{\text{Cs}}$, is exactly $9\,192\,631\,770\text{ Hz}$. So one second is the time taken by $9\,192\,631\,770$ periods of that radiation. 2019 Same meaning as the 1967 definition.
Electric current	Ampere	A	Fixed by the elementary charge: e is exactly $1.602\,176\,634 \times 10^{-19}\text{ C}$, where $\text{C} = \text{A s}$. So a current of 1 A carries about 6.24×10^{18} elementary charges every second. 2019 Earlier (1948–2019): defined by the force between two long, parallel current-carrying wires.
Thermodynamic temperature	Kelvin	K	Fixed by the Boltzmann constant: k is exactly $1.380\,649 \times 10^{-23}\text{ J K}^{-1}$ (here $1\text{ J} = 1\text{ kg m}^2\text{s}^{-2}$). 2019 Earlier (1967–2019): $\frac{1}{273.16}$ of the thermodynamic temperature of the triple point of water.
Amount of substance	Mole	mol	One mole contains exactly $6.022\,140\,76 \times 10^{23}$ elementary entities (atoms, molecules, ions, ...). This number is the Avogadro constant N_A, expressed in mol^{-1}. 2019 Earlier (1971–2019): as many entities as there are atoms in 0.012 kg of carbon-12.
Luminous intensity	Candela	cd	Fixed by the luminous efficacy K_{cd} of monochromatic light of frequency $540 \times 10^{12}\text{ Hz}$, which is exactly 683 lm W^{-1}. 2019 Same meaning as the 1979 definition (radiant intensity of $\frac{1}{683}$ watt per steradian).

What Changed in 2019?

Since **20 May 2019**, every SI base unit is defined using a **constant of nature**. Each constant is given an **exact, fixed value**. Now no unit depends on a physical object. Earlier, the kilogram was defined by a metal cylinder. This cylinder could get scratched or slowly change. Today the kilogram can be reproduced anywhere in the world from the Planck constant.

 $\Delta\nu_{Cs} \rightarrow \text{second}$
 $c \rightarrow \text{metre}$
 $h \rightarrow \text{kilogram}$
 $e \rightarrow \text{ampere}$
 $k \rightarrow \text{kelvin}$
 $N_A \rightarrow \text{mole}$
 $K_{cd} \rightarrow \text{candela}$

DEFINITION

Two Supplementary Quantities

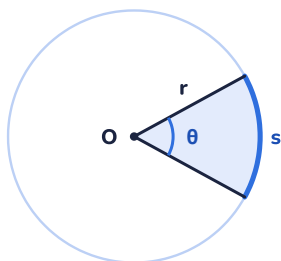
Apart from the seven base quantities, SI has **two supplementary quantities** for measuring angles. These are the **plane angle** and the **solid angle**.

Plane Angle

Radian · rad

One radian is the angle made at the **centre of a circle** by an arc whose length is equal to the **radius**.

$$d\theta = \frac{ds}{r}$$



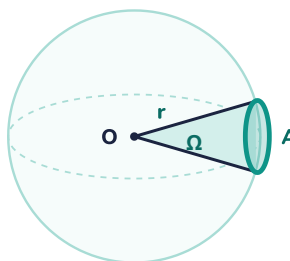
Arc s makes a plane angle θ at the centre O

Solid Angle

Steradian · sr

One steradian is the solid angle made at the **centre of a sphere** by a part of its surface whose area is equal to the **square of the radius**.

$$d\Omega = \frac{dA}{r^2}$$



Surface area A makes a solid angle Ω at the centre O

NOTE

- ◆ The **FPS system** is not a metric system. It is rarely used today.
- ◆ The **CGS system** has a weakness. Many of its derived units are **too small** to be convenient.
- ◆ The **MKS system** has an advantage. Many of its derived units are of a **practical size**.

1.6 SI Derived Units with Special Names

Many derived units have **special names**. Most of them are named in honour of **famous scientists**. The common ones are:

PHYSICAL QUANTITY	NAME	SYMBOL	IN TERMS OF OTHER UNITS	IN TERMS OF SI BASE UNITS
Frequency	Hertz	Hz	–	s^{-1}
Force	Newton	N	–	kg m s^{-2}
Pressure, stress	Pascal	Pa	N/m^2 or N m^{-2}	$\text{kg m}^{-1}\text{s}^{-2}$
Energy, work, quantity of heat	Joule	J	N m	$\text{kg m}^2\text{s}^{-2}$
Power, radiant flux	Watt	W	J/s or J s^{-1}	$\text{kg m}^2\text{s}^{-3}$
Quantity of electricity, electric charge	Coulomb	C	–	A s
Electric potential, potential difference, EMF	Volt	V	W/A or W A^{-1}	$\text{kg m}^2\text{s}^{-3}\text{A}^{-1}$
Capacitance	Farad	F	C/V	$\text{A}^2\text{s}^4\text{kg}^{-1}\text{m}^{-2}$
Electric resistance	Ohm	Ω	V/A	$\text{kg m}^2\text{s}^{-3}\text{A}^{-2}$
Conductance	Siemens	S	A/V	$\text{m}^{-2}\text{kg}^{-1}\text{s}^3\text{A}^2$
Magnetic flux	Weber	Wb	V s or J/A	$\text{kg m}^2\text{s}^{-2}\text{A}^{-1}$
Magnetic field, flux density, induction	Tesla	T	Wb/m^2	$\text{kg s}^{-2}\text{A}^{-1}$
Inductance	Henry	H	Wb/A	$\text{kg m}^2\text{s}^{-2}\text{A}^{-2}$
Luminous flux, luminous power	Lumen	lm	–	cd sr
Illuminance	Lux	lx	lm/m^2	$\text{m}^{-2}\text{cd sr}$
Activity (of a radio nuclide / radioactive source)	Becquerel	Bq	–	s^{-1}
Absorbed dose, absorbed dose index	Gray	Gy	J/kg	m^2s^{-2}

1.7 Some SI Derived Units Expressed by Means of SI Units

Other derived units have **no special name**. They are simply written as a **combination of SI units**. Useful examples:

PHYSICAL QUANTITY	NAME	SYMBOL	IN TERMS OF SI BASE UNITS
Magnetic moment	Joule per tesla	J T^{-1}	m^2A
Dipole moment	Coulomb metre	C m	s A m

PHYSICAL QUANTITY	NAME	SYMBOL	IN TERMS OF SI BASE UNITS
Dynamic viscosity	Poiseuille or pascal second or newton second per square metre	Pl or Pa s or N s/m ²	m ⁻¹ kg s ⁻¹
Torque, couple, moment of force	Newton metre	N m	m ² kg s ⁻²
Surface tension	Newton per metre	N/m	kg s ⁻²
Power density, irradiance, heat flux density	Watt per square metre	W/m ²	kg s ⁻³
Heat capacity, entropy	Joule per kelvin	J/K	m ² kg s ⁻² K ⁻¹
Specific heat capacity, specific entropy	Joule per kilogram kelvin	J/kg K	m ² s ⁻² K ⁻¹
Specific energy, latent heat	Joule per kilogram	J/kg	m ² s ⁻²
Radiant intensity	Watt per steradian	W sr ⁻¹	m ² kg s ⁻³ sr ⁻¹
Thermal conductivity	Watt per metre kelvin	W m ⁻¹ K ⁻¹	m kg s ⁻³ K ⁻¹
Energy density	Joule per cubic metre	J/m ³	kg m ⁻¹ s ⁻²
Electric field strength	Volt per metre	V/m	m kg s ⁻³ A ⁻¹
Electric charge density	Coulomb per cubic metre	C/m ³	m ⁻³ A s
Electric flux density	Coulomb per square metre	C/m ²	m ⁻² A s
Permittivity	Farad per metre	F/m	m ⁻³ kg ⁻¹ s ⁴ A ²
Permeability	Henry per metre	H/m	m kg s ⁻² A ⁻²
Molar energy	Joule per mole	J/mol	m ² kg s ⁻² mol ⁻¹
Angular momentum, Planck constant	Joule second	J s	kg m ² s ⁻¹
Molar entropy, molar heat capacity	Joule per mole kelvin	J/mol K	m ² kg s ⁻² K ⁻¹ mol ⁻¹
Exposure (X-rays and γ-rays)	Coulomb per kilogram	C/kg	kg ⁻¹ s A
Absorbed dose rate	Gray per second	Gy/s	m ² s ⁻³
Compressibility	Per pascal	Pa ⁻¹	m kg ⁻¹ s ²
Elastic moduli	Newton per square metre	N/m ² or N m ⁻²	kg m ⁻¹ s ⁻²
Pressure gradient	Pascal per metre	Pa/m or N m ⁻³	kg m ⁻² s ⁻²
Surface potential	Joule per kilogram	J/kg or N m/kg	m ² s ⁻²
Pressure energy	Pascal cubic metre	Pa m ³ or N m	kg m ² s ⁻²

PHYSICAL QUANTITY	NAME	SYMBOL	IN TERMS OF SI BASE UNITS
Impulse	Newton second	N s	kg m s^{-1}
Angular impulse	Newton metre second	N m s	$\text{kg m}^2 \text{s}^{-1}$
Specific resistance	Ohm metre	$\Omega \text{ m}$	$\text{kg m}^3 \text{s}^{-3} \text{A}^{-2}$
Surface energy	Joule per square metre	J/m^2 or N/m	kg s^{-2}

1.8 Advantages of SI over Other Systems of Units

i Coherent System

Every derived unit is obtained from the base units by **simple multiplication or division**. **No extra numerical factors** are needed.

ii Rational System

It has **only one unit for each quantity**. For example, all forms of energy are measured in **joule**.

iii Metric System

Larger and smaller units are formed using **powers of 10**, i.e. $a \times 10^{\pm b}$. This makes conversion between them easy.

iv Absolute System

It does **not use gravitational units**. So its definitions do not depend on the value of g .

1.9 Some General Units & Unit Conversion

• SOME GENERAL UNITS (OUTSIDE SI)

NAME	SYMBOL	VALUE IN SI UNIT
Minute	min	60 s
Hour	h	60 min = 3600 s
Day	d	24 h = 86400 s
Year	y	365.25 d = 3.156×10^7 s
Degree	°	$1^\circ = (\pi/180) \text{ rad}$
Litre	L	$1 \text{ dm}^3 = 10^{-3} \text{ m}^3$
Tonne	t	10^3 kg
Carat	c	200 mg
Bar	bar	$0.1 \text{ MPa} = 10^5 \text{ Pa}$
Curie	Ci	$3.7 \times 10^{10} \text{ s}^{-1}$
Roentgen	R	$2.58 \times 10^{-4} \text{ C/kg}$
Quintal	q	100 kg

NAME	SYMBOL	VALUE IN SI UNIT
Barn	b	$100 \text{ fm}^2 = 10^{-28} \text{ m}^2$
Are	a	$1 \text{ dam}^2 = 10^2 \text{ m}^2$
Hectare	ha	$1 \text{ hm}^2 = 10^4 \text{ m}^2$
Standard atmospheric pressure	atm	$101325 \text{ Pa} = 1.013 \times 10^5 \text{ Pa}$

PROBLEM SOLVING STRATEGY

Unit Conversion

1 Find what is needed.

Note the given units and the units asked for. First convert everything to SI units (metre, kilogram, second). This usually makes the problem easiest.

2 Write the conversion factors.

Units can be multiplied and divided like algebraic symbols. A factor such as $\frac{60 \text{ s}}{1 \text{ min}}$ is equal to 1. So multiplying by it changes the unit but not the quantity.

3 Multiply and cancel.

Multiply the quantity by the factors. Cancel the same units until only the required unit is left.

4 Check the result.

The answer should be in the unit asked for. The size of the number should be reasonable.

1.10 Rules for Writing SI Units

1 Rule 1

Unit **symbols** are normally written in **small (lower-case) letters**. Examples: m, kg, s.

2 Rule 2

A symbol is **never followed by a full stop**. Write "5 kg", not "5 kg." with a dot.

3 Rule 3

A symbol begins with a **capital letter** only when the unit is **named after a scientist**. Examples: N, Pa, J.

4 Rule 4

A unit written **in full** always begins with a **small letter**. This is true even if it is named after a scientist. Examples: newton, joule.

5 Rule 5

Symbols have **no plural form**. Write 10 kg, not 10 kgs.

1.11 Abbreviations in Powers of Ten

Very large and very small values are easier to write using **prefixes**. Each prefix stands for a **power of ten**. It gives a convenient **multiple or sub-multiple** of the SI unit:

MULTIPLE	PREFIX	SYMBOL	SUB-MULTIPLE	PREFIX	SYMBOL
10^1	deca	da	10^{-1}	deci	d
10^2	hecto	h	10^{-2}	centi	c
10^3	kilo	k	10^{-3}	milli	m
10^6	mega	M	10^{-6}	micro	μ
10^9	giga	G	10^{-9}	nano	n
10^{12}	tera	T	10^{-12}	pico	p
10^{15}	peta	P	10^{-15}	femto	f
10^{18}	exa	E	10^{-18}	atto	a

Examples

1 megaohm

$$M\Omega = 10^6 \Omega$$

1 milliamperere

$$1 \text{ mA} = 10^{-3} \text{ A}$$

1 kilometre

$$1 \text{ km} = 10^3 \text{ m}$$

1 microvolt

$$1 \mu\text{V} = 10^{-6} \text{ V}$$

1 decagram

$$1 \text{ dag} = 10 \text{ g}$$

1 nanosecond

$$1 \text{ ns} = 10^{-9} \text{ s}$$

1 centimetre

$$1 \text{ cm} = 10^{-2} \text{ m}$$

1 picofarad

$$1 \text{ pF} = 10^{-12} \text{ F}$$

NOTE

Why CGS, MKS and SI Are "Decimal" Systems

In CGS, MKS and SI, the larger and smaller units are related to the basic units by **powers of 10**. That is why they are called **metric** or **decimal systems** of units.

1.12 Some Important Practical Units

Some quantities are very **large or very small**. For them, special **practical units** are more convenient than SI units. They are grouped below by the quantity they measure:

For Length / Distance

ASTRONOMICAL / LARGE DISTANCES

Astronomical Unit (AU)

The average distance between the Earth and the Sun.

$$1 \text{ AU} = 1.496 \times 10^{11} \text{ m}$$

Light Year (ly)

The distance light travels in vacuum in one year.

$$1 \text{ ly} = 9.46 \times 10^{15} \text{ m}$$

Parallactic second (parsec)

The distance at which an arc of 1 AU makes an angle of one second of arc.

$$1 \text{ parsec} = 3.084 \times 10^{16} \text{ m} = 3.26 \text{ ly}$$

SUB-MICROSCOPIC / SMALL DISTANCES

Micron / micrometre

$$1 \mu\text{m} = 10^{-6} \text{ m}$$

Nanometre

$$1 \text{ nm} = 10^{-9} \text{ m}$$

Angstrom unit ($\text{\AA}$)

$$1 \text{ \AA} = 10^{-10} \text{ m}$$

Fermi

Used for the sizes of nuclei.

$$1 \text{ fm} = 10^{-15} \text{ m}$$

For Mass

CONVENTIONAL / LARGE MASSES

Pound

$$1 \text{ lb} = 0.4536 \text{ kg}$$

Slug

$$1 \text{ slug} = 14.59 \text{ kg}$$

Quintal

$$1 \text{ q} = 100 \text{ kg}$$

Tonne / Metric ton

$$1 \text{ t} = 1000 \text{ kg}$$

ATOMIC SCALE

Atomic mass unit (amu)

One-twelfth ($\frac{1}{12}$) of the mass of one ${}^6\text{C}^{12}$ atom.

$$1 \text{ amu} = 1.66 \times 10^{-27} \text{ kg}$$

For Time

ASTRONOMICAL & CALENDAR UNITS

Solar day

The time the Earth takes to rotate once on its own axis, measured with respect to the Sun.

Sidereal day

The time the Earth takes to rotate once on its own axis, measured with respect to a distant star.

Solar year

The time the Earth takes to go once around the Sun in its orbit. 1 solar year = 365.25 average solar days = 366.25 sidereal days.

Tropical year

The time from one spring (vernal) equinox to the next. It is about 365.24 days. Our calendar year is based on it.

Leap year

A year divisible by 4, in which February has 29 days. A century year must be divisible by 400.

Lunar month

The time the Moon takes to go once around the Earth.

$$1 \text{ lunar month} = 27.3 \text{ days}$$

SMALLEST PRACTICAL UNIT

Shake

The smallest practical unit of time.

$$1 \text{ shake} = 10^{-8} \text{ s}$$

For Areas

Barn

$$1 \text{ barn} = 10^{-28} \text{ m}^2$$

Acre

$$1 \text{ acre} = 4047 \text{ m}^2$$

Hectare

$$1 \text{ hectare} = 10^4 \text{ m}^2$$

For Other Quantities

VOLUME

Litre

Here cc means cubic centimetre (cm^3).

$$1 \text{ L} = 10^3 \text{ cc} = 10^{-3} \text{ m}^3$$

Gallon

USA: 1 gallon = 3.7854 L; UK: 1 gallon = 4.546 L.

PRESSURE

Pascal

Atmospheric pressure is about $1 \text{ atm} = 1.01 \times 10^5 \text{ Pa}$.

$$1 \text{ Pa} = 1 \text{ N m}^{-2}$$

Bar

Very close to atmospheric pressure ($1 \text{ atm} \approx 1.013 \text{ bar}$).

$$1 \text{ bar} = 10^5 \text{ Pa}$$

Torr

The pressure of a 1 mm column of mercury.

$$1 \text{ torr} = 133.3 \text{ Pa}$$

ENERGY / WORK

Electron volt

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

Erg

$$1 \text{ erg} = 10^{-7} \text{ J}$$

Kilowatt hour

$$1 \text{ kWh} = 3.6 \times 10^6 \text{ J}$$

POWER, OPTICS & ANGLE

Horse power

$$1 \text{ HP} = 746 \text{ W}$$

Dioptre

Unit of the power of a lens.

$$1 \text{ D} = 1 \text{ m}^{-1}$$

Degree

Unit of plane angle.

$$1^\circ = (\pi/180) \text{ rad}$$

02

PART 2

Significant Figures & Rounding Off

DEFINITION

Significant Figures

Every measured value has some digits that are **certain**. They are followed by one last digit, which is only an **estimate**. The reliable digits and this **first uncertain digit** together are called the **significant figures** (or significant digits) of the measurement.

Reading a Measurement • 374.5 m



= 4 significant figures

The digits 3, 7 and 4 are **certain**. The last digit, 5, is **estimated**. Any more digits would add **no real information**.

2.1 Rules for Determining Significant Figures

☒ significant ☐ not significant

1 Rule 1

Every **non-zero digit** is significant.

1 2 3 4 → 4 S.F.

2 Rule 2

Zeros between non-zero digits are always significant. It does not matter where the decimal point is, or if there is none.

1 0 0 7 → 4 S.F.

1 0 0 . 7 → 4 S.F.

3 Rule 3

In a number **smaller than one**, the zeros after the decimal point but **before the first non-zero digit** are not significant.

0 . 0 0 5 7 0 4 → 4 S.F.

4 Rule 4

In a number **without a decimal point**, the **trailing zeros** (zeros at the end) are not significant.

3 2 1 0 → 3 S.F.

5 Rule 5

In a number **with a decimal point**, the **trailing zeros** are significant.

3 . 5 0 0 → 4 S.F.

6 Rule 6

Exact numbers have **unlimited** significant figures. They are pure numbers used to multiply or divide. They are not measured or rounded.

e.g. In $r = \frac{d}{2}$ and $s = 2\pi r$, the factor 2 is exact. It can be treated as 2.0, 2.00, 2.000 and so on.

2 → ∞ S.F. (exact)

7 Rule 7

Changing the **unit** never changes the number of significant figures. For example, 16.4 cm, 0.164 m and 0.000164 km all have **three**.

1 6 . 4 cm → 3 S.F. 0 . 1 6 4 m → 3 S.F.

0 . 0 0 0 1 6 4 km → 3 S.F.

2.2 Ambiguity & Scientific Notation

REMEMBER

Ambiguity in Significant Figures

Trailing zeros can cause confusion. Suppose a length is measured as $x = 4.700$ m. Here the zeros show the **precision** of the measurement. So they are **significant**. The same length can also be written as $x = 0.004700$ km, $x = 470.0$ cm or $x = 4700$ mm.

By Rule 4, 4700 mm would **wrongly** seem to have only two significant figures. But 0.004700 km clearly shows four. **Changing units cannot change** the number of significant figures (Rule 7). So we need a clearer way to write such values.

The Same Length, Four Ways · $x = 4.700$ m

4 . 7 0 0 m → 4 S.F.

0 . 0 0 4 7 0 0 km → 4 S.F.

4 7 0 . 0 cm → 4 S.F.

4 7 0 0 mm → looks like 2 S.F.? – ambiguous!

The length is the same in every line. But **4700 mm** seems to have only two significant figures. The solution:

Write every measurement in **scientific notation**, i.e. in powers of 10. Any number can be written in the form:

SCIENTIFIC NOTATION

$$a \times 10^b$$

a = a number between 1 and 10, b = a positive or negative power of 10 (the **order of magnitude**)

e.g. $x = 4.700 \text{ m} = 4.700 \times 10^{-3} \text{ km}$ has **4** significant figures. The power of 10 is not counted.

2.3 Rounding Off

A result calculated from approximate values often has too many uncertain digits. So it is **rounded off**. The usual rules are:

1 Rule 1

If the digit to be dropped is **less than 5**, keep the digit before it **unchanged**.

$$7.8\cancel{2} \rightarrow 7.8$$

2 Rule 2

If the digit to be dropped is **more than 5**, **increase** the digit before it **by one**.

$$6.8\cancel{7} \rightarrow 6.9$$

3 Rule 3

If the digit to be dropped is **5 followed by non-zero digits**, **increase** the digit before it **by one**.

$$16.3\cancel{54} \rightarrow 16.4$$

4 Rule 4

If the digit to be dropped is **5, or 5 followed only by zeros**, keep the digit before it **unchanged if it is even**.

$$3.2\cancel{50} \rightarrow 3.2 \quad (2 \text{ is even})$$

5 Rule 5

If the digit to be dropped is **5, or 5 followed only by zeros**, **increase** the digit before it **by one if it is odd**.

$$3.\cancel{750} \rightarrow 3.8 \quad (7 \text{ is odd})$$

2.4 Rules for Arithmetical Operations with Significant Figures

Measured values that we add, subtract, multiply or divide may have **different precisions**. The answer can **never be more precise** than the least precise value used. So we round it using these rules:

Addition and Subtraction

Keep as many **decimal places** as the value that has the **fewest decimal places**.

RULE Round the answer to the fewest decimal places

$$\underset{1 \text{ d.p.}}{2.1} + 1.78 + 2.046 = 5.926 \text{ m}$$

round → **5.9 m** 1 d.p. (set by 2.1)

Similarly, for $x = 12.587$ m and $y = 12.5$ m:
1 d.p.

$$x - y = 12.587 - 12.5 = 0.087 \text{ m}$$

round → **0.1 m** 1 d.p. (set by 12.5)

Multiplication and Division

Keep as many **significant figures** as the value that has the **fewest significant figures**.

RULE Round the answer to the **fewest significant figures**

e.g. The speed of light is 3.00×10^8 m/s. One year is 3.1557×10^7 s. So one light year is:
3 S.F.

$$3.00 \times 10^8 \times 3.1557 \times 10^7 = 9.4671 \times 10^{15} \text{ m}$$

round → **9.47×10^{15} m** 3 S.F. (set by 3.00)

2.5 Rules for Determining Uncertainty in the Results of Arithmetic Calculations

1 Rule 1 • Product (Combining Percentage Errors)

A thin rectangular sheet is 15.4 cm long and 10.2 cm wide. Both values have **three significant figures**. They are precise to the first decimal place (± 0.1 cm).

1 Convert each **absolute error** into a **percentage error**:

$$l = (15.4 \pm 0.1) \text{ cm} = 15.4 \text{ cm} \pm \left(\frac{0.1}{15.4} \times 100 \right) \% = 15.4 \text{ cm} \pm 0.6\%$$

$$b = (10.2 \pm 0.1) \text{ cm} = 10.2 \text{ cm} \pm \left(\frac{0.1}{10.2} \times 100 \right) \% = 10.2 \text{ cm} \pm 1\%$$

2 For a **product**, the percentage errors **are added**:

$$A = 15.4 \times 10.2 \pm (0.6 + 1.0)\% = 157.08 \text{ cm}^2 \pm 1.6\% = (157.08 \pm 2.51) \text{ cm}^2$$

3 Keep **3 S.F.** in the area and **1 S.F.** in its error:

$$A = (157 \pm 3) \text{ cm}^2$$

2 Rule 2 • Result Retains the Significant Figures of the Data

If the data has n **significant figures**, the result can be trusted only up to n **significant figures**.

e.g. $x = 13.7$ m and $y = 8.08$ m each have **three significant figures**:

$$x - y = 13.7 - 8.08 = 5.62 \text{ m}$$

round $\rightarrow$ **5.6 m** keep the fewest decimal places (1 d.p., set by 13.7)

3 Rule 3 • Relative Error Depends on the Number Itself

The **relative error** of a value depends on its number of significant figures. It also depends on the **size of the number**. For example, $m_1 = (1.04 \pm 0.01)$ kg and $m_2 = (9.24 \pm 0.01)$ kg have the **same absolute error** of 0.01 kg:

$$1.04 \text{ kg} \rightarrow \pm \frac{0.01}{1.04} \times 100 = \pm 1\%$$

$$9.24 \text{ kg} \rightarrow \pm \frac{0.01}{9.24} \times 100 = \pm 0.1\%$$

INSIGHT Larger value $\Rightarrow$ smaller relative error

4 Rule 4 • Keep One Extra Digit in Multi-Step Calculations

In a calculation with several steps, round off only at the end. In the middle steps, keep **one more significant figure** than the least precise measurement has.

e.g. Start with $x = 9.58$. Find $\frac{1}{x}$. Then take the reciprocal again. Do we get 9.58 back?

✗ KEEP 3 DIGITS

$$\frac{1}{9.58} = 0.104$$

$$\frac{1}{0.104} = 9.62$$

$\neq 9.58$ — an error appears

✓ KEEP 4 DIGITS

$$\frac{1}{9.58} = 0.1044$$

$$\frac{1}{0.1044} = 9.58$$

$= 9.58$ — correct

Keeping **one extra digit** in the middle steps stops rounding errors from growing.

03

PART 3

Dimensions of a Physical Quantity

DEFINITION

Dimensions of a Physical Quantity

The **dimensions** of a physical quantity are the **powers** to which the base quantities must be raised to express that quantity. Each of the **seven base quantities** has its own dimension symbol. It is written inside square brackets []:

[L] Length

[M] Mass

[T] Time

[A] Electric current

[K] Temperature

[cd] Luminous intensity

[mol] Amount of substance

Example • Volume

Volume is **length** × **length** × **length**:

$$V = [L] \times [L] \times [L] = [L^3]$$

It has no mass and no time. So $V = [M^0 L^3 T^0]$.

Example • Force

Force is **mass** × **acceleration**:

$$F = \text{mass} \times \text{acceleration} = \text{mass} \times \frac{\text{length}}{(\text{time})^2}$$

$$\therefore F = \frac{[M][L]}{[T^2]} = [MLT^{-2}]$$

NOTE

Square brackets [] around a quantity show its **dimensions**, not its value.

3.1 Dimensional Formulae & Dimensional Equations

DEFINITION

Dimensional Formula

An expression that shows the **base quantities** in a physical quantity and their **powers** is called its **dimensional formula**.

e.g. powers colour-coded by base quantity — **M** mass, **L** length, **T** time:

Acceleration

$$[M^0 L^1 T^{-2}]$$

Mass density

$$[M^1 L^{-3} T^0]$$

Volume

$$[M^0 L^3 T^0]$$

DEFINITION

Dimensional Equation

When a physical quantity is written **equal to its dimensional formula**, we get its **dimensional equation**.

e.g. each line shows the quantity, how it is defined, and its dimensions:

Linear momentum = mass $\times$ velocity = $[M^1L^1T^{-1}]$

Impulse = force $\times$ time = $[M^1L^1T^{-1}]$

Moment of inertia = mass $\times$ (radius of gyration)² = $[M^1L^2T^0]$

PROBLEM SOLVING STRATEGY

Finding Dimensional Formulae

- 1 Read the question and note whether a **formula** or a **law** for the quantity is given.
- 2 Write down the **defining formula** of the quantity whose dimensions you need.
- 3 Write that formula in terms of **fundamental quantities**, e.g. acceleration = $\frac{\text{Length}}{\text{Time}^2}$.
- 4 Replace each fundamental quantity by its **symbol**: mass = $[M]$, length = $[L]$, time = $[T]$, and so on.
- 5 **Simplify** the powers algebraically to get the result.
- 6 Write the answer in the standard order $[M], [L], [T] \dots$

3.2 Dimensional Formulae of Some Important Quantities

Dimensional formulae are colour-coded by base quantity — **M** mass, **L** length, **T** time, **A** current.

PHYSICAL QUANTITY	RELATION WITH OTHER QUANTITIES	DIMENSIONAL FORMULA	SI UNIT
Area	length $\times$ breadth	$[M^0L^2T^0]$	m^2
Volume	length $\times$ breadth $\times$ height	$[M^0L^3T^0]$	m^3
Density	mass / volume	$[M^1L^{-3}T^0]$	$kg\ m^{-3}$
Specific gravity	density of body / density of water at 4°C	$[M^0L^0T^0]$ (dimensionless)	no units
Speed or velocity	distance / time	$[M^0L^1T^{-1}]$	$m\ s^{-1}$
Linear momentum	mass $\times$ velocity	$[M^1L^1T^{-1}]$	$kg\ m\ s^{-1}$
Acceleration	change in velocity / time	$[M^0L^1T^{-2}]$	$m\ s^{-2}$
Acceleration due to gravity (g)	change in velocity / time	$[M^0L^1T^{-2}]$	$m\ s^{-2}$

PHYSICAL QUANTITY	RELATION WITH OTHER QUANTITIES	DIMENSIONAL FORMULA	SI UNIT
Force	mass $\times$ acceleration	$[M^1L^1T^{-2}]$	N (newton)
Impulse	force $\times$ time	$[M^1L^1T^{-1}]$	N s
Pressure	force / area	$[M^1L^{-1}T^{-2}]$	N m ⁻²
Universal constant of gravitation (G)	$G = \frac{Fr^2}{m_1m_2}$	$[M^{-1}L^3T^{-2}]$	N m ² kg ⁻²
Work	force $\times$ distance	$[M^1L^2T^{-2}]$	J (joule)
Energy (all types)	work	$[M^1L^2T^{-2}]$	J (joule)
Moment of force	force $\times$ distance	$[M^1L^2T^{-2}]$	N m
Power	work / time	$[M^1L^2T^{-3}]$	W (watt)
Surface tension	force / length	$[M^1L^0T^{-2}]$	N m ⁻¹
Surface energy	energy / area	$[M^1L^0T^{-2}]$	J m ⁻²
Force constant	force / displacement	$[M^1L^0T^{-2}]$	N m ⁻¹
Thrust	force	$[M^1L^1T^{-2}]$	N (newton)
Tension	force	$[M^1L^1T^{-2}]$	N (newton)
Stress, pressure	force / area	$[M^1L^{-1}T^{-2}]$	N m ⁻²
Strain	change in dimension / original dimension	$[M^0L^0T^0]$	no units
Coefficient of elasticity	stress / strain	$[M^1L^{-1}T^{-2}]$	N m ⁻²
Radius of gyration (K)	distance	$[M^0L^1T^0]$	m
Moment of inertia (I)	mass $\times$ (radius of gyration) ²	$[M^1L^2T^0]$	kg m ²
Angle / angular displacement (θ)	arc / radius	$[M^0L^0T^0]$	radian
Angular velocity (ω)	angle / time	$[M^0L^0T^{-1}]$	rad s ⁻¹
Angular acceleration (α)	change in angular velocity / time	$[M^0L^0T^{-2}]$	rad s ⁻²
Angular momentum	$I\omega$	$[M^1L^2T^{-1}]$	kg m ² s ⁻¹
Torque	force $\times$ distance	$[M^1L^2T^{-2}]$	N m
Wavelength (λ)	length of one wave (distance)	$[M^0L^1T^0]$	m
Frequency (ν)	number of vibrations / time	$[M^0L^0T^{-1}]$	s ⁻¹ or Hz
Angular frequency (ω)	$2\pi \times$ frequency	$[M^0L^0T^{-1}]$	rad s ⁻¹
Velocity of light in vacuum (c)	distance / time	$[M^0L^1T^{-1}]$	m s ⁻¹
Velocity gradient	velocity / distance	$[M^0L^0T^{-1}]$	s ⁻¹

PHYSICAL QUANTITY	RELATION WITH OTHER QUANTITIES	DIMENSIONAL FORMULA	SI UNIT
Rate of flow	volume / time	$[M^0 L^3 T^{-1}]$	$m^3 s^{-1}$
Planck's constant (h)	energy / frequency	$[M^1 L^2 T^{-1}]$	J s
Linear mass density (m)	mass / length	$[M^1 L^{-1} T^0]$	$kg\ m^{-1}$
Distance travelled in n th second	distance / time	$[M^0 L^1 T^{-1}]$	$m\ s^{-1}$
Avogadro's number (N)	number of molecules per mole	$[M^0 L^0 T^0]$	mol^{-1}
Magnetic dipole moment (M)	$I \times A$ (current $\times$ area)	$[M^0 L^2 T^0 A^1]$	$A\ m^2$
Pole strength (m)	$m = \frac{M}{2l}$	$[M^0 L^1 T^0 A^1]$	A m
Magnetic permeability of free space (μ_0)	$\mu_0 = \frac{4\pi F r^2}{m_1 m_2}$	$[M^1 L^1 T^{-2} A^{-2}]$	$H\ m^{-1}$
Resistance (R)	potential difference / current	$[M^1 L^2 T^{-3} A^{-2}]$	Ω (ohm)
Capacitance (C)	charge / potential difference	$[M^{-1} L^{-2} T^4 A^2]$	F (farad)
Surface density of charge	charge / area	$[M^0 L^{-2} T^1 A^1]$	$C\ m^{-2}$
Electric dipole moment (p)	$q(2d)$	$[M^0 L^1 T^1 A^1]$	C m
Specific resistance / resistivity (ρ)	$\frac{RA}{l}$	$[M^1 L^3 T^{-3} A^{-2}]$	$\Omega\ m$
Conductance (G)	$1/R$	$[M^{-1} L^{-2} T^3 A^2]$	S (siemens)
Conductivity (σ)	$1/\rho$	$[M^{-1} L^{-3} T^3 A^2]$	$S\ m^{-1}$
Electric flux	electric field $\times$ area	$[M^1 L^3 T^{-3} A^{-1}]$	$N\ m^2 C^{-1}$
Faraday constant	Avogadro number $\times$ elementary charge	$[M^0 L^0 T^1 A^1]$	$C\ mol^{-1}$
Mass defect (Δm)	sum of masses of nucleons – mass of nucleus	$[M^1 L^0 T^0]$	kg
Binding energy of nucleus	(mass defect) $\times$ (speed of light) ²	$[M^1 L^2 T^{-2}]$	J (joule)

3.3 Dimensional Analysis and Its Applications

Dimensional analysis is a quick and powerful tool. It is used to **check equations** for consistency (homogeneity). It can also **find relations** between quantities. It has three main uses:

1

Checking whether an equation is **dimensionally consistent**

SEE 3.4

2

Converting a value from one system of units to another

SEE 3.5

3

Finding the relation between physical quantities

SEE 3.6

3.4 Checking the Dimensional Consistency of Equations

Only quantities with the **same dimensions** can be **added or subtracted**. For example, a mass cannot be added to a velocity. A current cannot be subtracted from a time. This simple idea gives the **principle of homogeneity of dimensions**. We use it to test equations.

PRINCIPLE

Principle of Homogeneity of Dimensions

An equation is **dimensionally correct** only if **every term** on **both sides** has the **same dimensions**.

e.g. Test the equation of motion $s = ut + \frac{1}{2}at^2$. Each term must have the dimensions $[L]$:

$$[s] = [L] \quad \checkmark$$

$$[ut] = [LT^{-1}][T] = [L] \quad \checkmark$$

$$[\frac{1}{2}at^2] = [LT^{-2}][T^2] = [L] \quad \checkmark$$

RESULT Every term reduces to $[L]$. So the equation is **dimensionally correct**

3.5 Conversion of One System of Units into Another

A quantity has the **same size** in every system of units. Its numerical value is **inversely proportional** to the size of the unit. So $n_1u_1 = n_2u_2$, which gives $n_2 = \frac{n_1u_1}{u_2}$. Let the two systems have base units M_1, L_1, T_1 and M_2, L_2, T_2 . Let the quantity have dimensions $[M^aL^bT^c]$. Then:

CONVERSION BETWEEN SYSTEMS

$$n_2 = n_1 \left[\frac{M_1}{M_2} \right]^a \left[\frac{L_1}{L_2} \right]^b \left[\frac{T_1}{T_2} \right]^c$$

n_1, n_2 = numerical values in the two systems; a, b, c = dimensions of the quantity in mass, length and time

3.6 Deducing the Relation among the Physical Quantities

Suppose we know **which quantities** something depends on. Then dimensions can tell us **how** it depends on them. Take a **simple pendulum**. Its **time period** t may depend on its length (l), the mass of the bob (m) and the acceleration due to gravity (g).

DERIVATION

SETUP

Assume $t = k m^a l^b g^c \dots (i)$. Here k is a **dimensionless constant**, and a, b, c are the powers to be found.

- 1 Write both sides in terms of M, L and T:

$$[M^0 L^0 T^1] = M^a L^b [LT^{-2}]^c = M^a L^{b+c} T^{-2c}$$

- 2 By the principle of homogeneity, equate the powers of M, L and T on both sides:

$$a = 0, \quad -2c = 1 \Rightarrow c = -\frac{1}{2}, \quad b + c = 0 \Rightarrow b = \frac{1}{2}$$

- 3 Put these values of a, b, c back into Eq. (i):

$$t = k m^0 l^{1/2} g^{-1/2} = k \sqrt{\frac{l}{g}}$$

TIME PERIOD OF A SIMPLE PENDULUM

$$t = k \sqrt{\frac{l}{g}}$$

k = dimensionless constant (experiments give $k = 2\pi$); dimensional analysis alone cannot find k

PROBLEM SOLVING STRATEGY

Deriving an Expression

Remember: dimensions can show that an equation is **dimensionally valid**. But they cannot prove the **exact** relation between the quantities.

- 1 Read the problem carefully. Understand the physics involved.
- 2 List the **known** and the **unknown** quantities.
- 3 Decide which **physical principle** connects these quantities.
- 4 Write the **equation** that links them. It must agree with the principle you chose.
- 5 Solve for the unknowns **algebraically** first. Substitute numbers only at the next step (except where some terms are zero).
- 6 Substitute the known values **along with their units**. This gives a numerical answer with units.
- 7 **Check your answer**. Are the units right? Is the size of the answer sensible? Compare it with a rough estimate.

Limitation 1

It cannot tell us whether a quantity is a **scalar or a vector**.

Limitation 2

It gives **no value for dimensionless constants** in a formula, such as 1, 2 or π .

Limitation 3

It cannot derive formulas that contain **trigonometric, logarithmic or exponential** functions. These functions have no dimensions.

Limitation 4

The method fails if a quantity depends on **more than three** quantities that have dimensions. This is because comparing the powers of M, L and T gives only **three equations**.