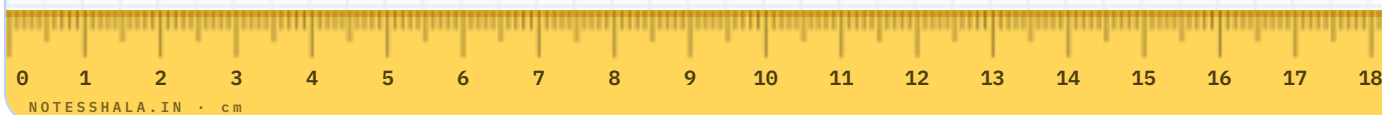


CHAPTER

14

# Waves

Waves and Their Types • Progressive Waves and Speed • Superposition and Standing Waves • Organ Pipes and Beats



## What's inside

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READING KEY

DEFINITION

NOTE

REMEMBER

DERIVATION

STRATEGY

Formulas sit on graph paper • coloured words are the key terms

# 01

## PART 1

# Waves and Their Types

A wave carries **energy** from one place to another. The material of the medium does **not** travel along with it. All our **communication** depends on signals that travel as waves.

## 1.1 Wave Motion

### DEFINITION

A **wave** is a **disturbance** that moves from one place to another through a medium or through vacuum. It is usually **periodic**. It carries **energy and momentum**, but there is **no bulk motion** of the medium.

A wave starts when a system is pushed away from its **equilibrium** (normal) state. This disturbance then **spreads** from one part of the system to the other parts.

**System disturbed → Disturbance spreads → Energy and information travel → Matter stays in place**

### REMEMBER

A wave carries **information** and **energy**, as signals, from one point to another. The **matter of the medium does not move** along with the wave as a whole.

## 1.2 Types of Waves

We usually study **three types** of waves.

### i Mechanical Waves

Waves that need a **material medium** to travel are called **mechanical waves**.

They are also called **elastic waves**. The medium must have **elasticity** and **inertia**.

They obey **Newton's laws of motion**. They exist only in a medium like water, air, rock or a string.

**e.g.** Water waves, sound waves and seismic waves (energy waves through the Earth's layers).

### ii Electromagnetic Waves

Waves made of **oscillating electric and magnetic fields** are called **electromagnetic waves**.

They **need no medium** to travel. So they are also called **non-mechanical waves**.

**e.g.** Visible light, ultraviolet light, X-rays, radio and TV waves, microwaves.

### iii Matter Waves

Waves linked with **moving microscopic particles** are called **matter waves** or **de Broglie waves**.

The particles can be electrons, protons, other fundamental particles, or even atoms and molecules.

**e.g.** An **electron microscope** uses the matter waves of moving electrons.

#### SPEED OF ELECTROMAGNETIC WAVES IN VACUUM

$$c = 299\,792\,458 \text{ m s}^{-1} \approx 3 \times 10^8 \text{ m s}^{-1}$$

All electromagnetic waves travel through vacuum with this **same speed**  $c$ , the speed of light.

#### NOTE

#### Dual Nature of Light

- ◆ Light is an **electromagnetic wave**. All the radiation we get from the **Sun** is electromagnetic.
- ◆ Light shows a **dual nature**. It behaves both as a **wave** and as a **particle**.
- ◆ In the same way, a **moving particle** can behave like a wave. This wave is its **matter wave**.

## 1.3 Characteristics of Wave Motion

- 1 The particles of the medium **oscillate again and again** about their **mean positions**. This passes the disturbance through the medium.
- 2 **Energy** moves from one place to another. The **particles themselves do not travel** with the wave.
- 3 Each particle gets the disturbance **a little later** than the particle before it. So there is a **continuous phase difference** between neighbouring particles.
- 4 The **wave velocity** is not the same as the **particle velocity**. Particle velocity is the velocity of each particle in its SHM about the mean position.
- 5 In a given medium, the **wave velocity stays constant**. But the **particle velocity keeps changing** during each oscillation.

Particle velocity at the **mean position**

Maximum

Particle velocity at an **extreme position**

Zero

A medium can carry a **mechanical wave** only if it has these properties:

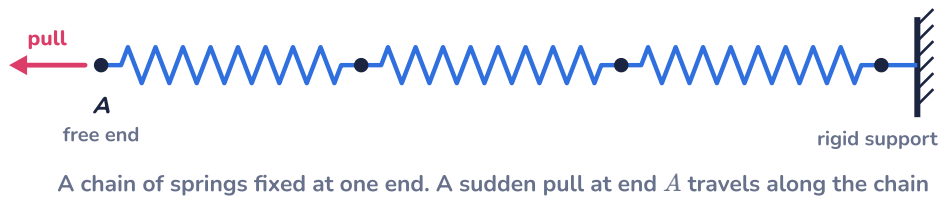
1 Inertia

2 Elasticity

3 Very low friction

## 1.4 Spring Model of Wave Propagation

Take a chain of **springs** joined end to end. One end is fixed to a **rigid support**. Pull the spring at the free end suddenly and let it go. The **disturbance travels** to the other end.



### • WHY THE DISTURBANCE TRAVELS

- 1 The first spring is pulled, so it gets **stretched**.
- 2 Due to **elasticity**, a **restoring force** develops in it.
- 3 This force brings the first spring back to its **mean position**. At the same time, it stretches the **second spring**.
- 4 The second spring then acts on the third, and so on.

**Spring pulled and released → Restoring force → Next spring stretched → Disturbance travels**

### KEY

The disturbance reaches the far end. But every spring just oscillates about its own equilibrium position.

## 1.5 Sound Waves in Air and in Solids

### • SOUND WAVES IN AIR

Think of a **small region of air** as a spring. It is joined to the regions around it, like springs in a chain. A sound wave **compresses** and **expands** each small region of air. This changes the **density** and the **pressure** of that region.

By **Boyle's law** (at constant temperature), pressure is inversely proportional to volume. So pressure is proportional to density:

$$p \propto \frac{1}{V} \Rightarrow p \propto \rho \Rightarrow (\Delta p) \propto (\Delta \rho)$$

Pressure is **force per unit area**. So a **restoring force** develops. It is proportional to the change in density. This is just like a stretched or compressed spring.

### i A Region Is Compressed

Its molecules move out into the **next region**. The density there increases. So a **compression** forms in the next region. The first region now has lower density. It becomes a **rarefaction**.

### ii A Region Is Rarefied

Air from the surroundings **rushes in**. So the **rarefaction moves** to the next region.

KEY

**Compressions and rarefactions pass from one region to the next. So a disturbance can travel through air.**

### • SOUND WAVES IN A SOLID

In a **crystalline solid**, think of the **atoms** as end points. **Springs** join each pair of neighbouring atoms.

The forces from the surrounding atoms **cancel out**. So each atom is in **equilibrium**.

A sound wave **displaces** an atom from its equilibrium position. A **restoring force** develops. The disturbance is handed on to the **neighbouring atom**, then to the one after it.

**Atom pushed from its place → Restoring force acts → Neighbour atom moves → Sound passes through the solid**

## 1.6 Transverse Waves

### DEFINITION

A **transverse wave** is a wave in which the particles of the medium do **SHM** about their mean positions **perpendicular** to the direction of travel of the wave.

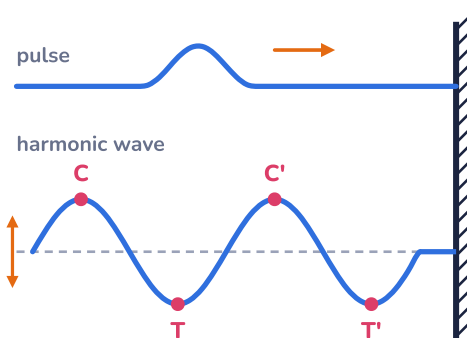
e.g. Waves on a string, ripples on the surface of water, and the vibrating strings of a **sitar** or a **guitar**.

### • ILLUSTRATION: WAVE ON A STRING

Take a long **stretched string**. Hold one end in your hand. The other end is fixed to a **rigid support**.

Give the free end a **sudden upward jerk**. An upward **kink**, or **pulse**, forms there. It travels along the string towards the fixed end.

Now move the free end **up and down continuously**. A train of **sinusoidal waves** travels along the string. These waves are **transverse**.



A single pulse (top) and a harmonic wave (bottom) on a stretched string

### i Crest

Points  $C, C', \dots$  are **crests**. A crest is a point of **maximum displacement** in the **positive (upward)** direction.

### ii Trough

Points  $T, T', \dots$  are **troughs**. A trough is a point of **maximum displacement** in the **negative (downward)** direction.

#### • MEDIA FOR TRANSVERSE WAVES

In a transverse wave, the particles move **perpendicular** to the wave. So each small part of the medium suffers a **shearing strain**. A transverse wave can travel only in a medium that can bear a **shearing stress**.

**Solids** and **strings** can bear shearing stress

Transverse waves possible

**Fluids** cannot bear shearing stress

Transverse waves not possible

## 1.7 Longitudinal Waves

### DEFINITION

A **longitudinal wave** is a wave in which the particles of the medium do **SHM** about their mean positions **along** the direction of travel of the wave.

e.g. Waves in a spring, sound waves, and the vibrating air column in **organ pipes**.

#### • ILLUSTRATION: PISTON IN AN AIR-FILLED PIPE

Take a **long pipe filled with air**. A **piston** is fitted at one end,  $A$ .



Moving the piston back and forth sends compressions (C) and rarefactions (R) along the pipe

### i Piston Pushed In

The **layer of air** next to the piston gets **compressed**. This layer moves right and compresses the next layer, and so on. So a **pulse of compression** reaches the other end. The air motion and the pressure change travel together as a pulse.

### ii Piston Pulled Back

The layer next to the piston becomes **rarefied**, so its **pressure falls**. Air from the next layer moves in to make up the pressure. Now that layer becomes rarefied. So a **pulse of rarefaction** travels to the right.

Now push and pull the piston **again and again in SHM**. A **sinusoidal wave** travels along the pipe. It is a series of **compressions (C)** and **rarefactions (R)**.

**KEY** The air layers oscillate parallel to the direction of the wave. So sound waves in air are longitudinal.

### Compression

A **compression** is a region where neighbouring particles are **closer than normal**.

- ◆ Volume **decreases**
- ◆ Density **increases**

### Rarefaction

A **rarefaction** is a region where neighbouring particles are **farther apart than normal**.

- ◆ Volume **increases**
- ◆ Density **decreases**

Both changes are temporary. They last only for a short time, while the wave passes.

## 1.8 Media for Longitudinal Waves

**KEY** Solids and fluids can all bear a compressive strain. So longitudinal waves can travel in all elastic media.

Some facts about the medium and the waves in it:

A **steel bar** can carry

Both transverse and longitudinal waves

**Air** can carry

Only longitudinal waves

- ◆ In the **same medium**, transverse and longitudinal waves travel with **different speeds**.
- ◆ In wave motion, **only the disturbance travels**. The **medium does not travel** with it.
- ◆ Waves can be **one-, two- or three-dimensional**. This depends on whether the energy spreads in one, two or three dimensions.

### 1-D Wave

Transverse waves along a **string**.

### 2-D Wave

**Ripples** on the surface of water.

### 3-D Wave

**Sound waves** from a point source.

## 02 PART 2 Progressive Waves and Speed

This part writes a travelling wave as an **equation**. It then finds the **speed of waves** on a string and the speed of sound in solids, liquids and gases.

## 2.1 Equation of a Progressive Wave

To describe a travelling wave, we need the **position**  $x$  and the **time**  $t$ . Take a wave moving in the **positive**  $x$ -**direction**. Let  $y(x, t)$  be the **transverse displacement** of the element at position  $x$  at time  $t$ . Then:

DISPLACEMENT RELATION (WAVE ALONG +X) ... (I)

$$y(x, t) = a \sin(kx - \omega t + \phi)$$

The same wave can also be written as a **sum of a sine term and a cosine term**:

$$y(x, t) = A \sin(kx - \omega t) + B \cos(kx - \omega t)$$

Here,

$$a = \sqrt{A^2 + B^2} \quad \text{and} \quad \phi = \tan^{-1} \left( \frac{B}{A} \right)$$

**KEY** The equation shows that the particles of the medium at every position execute SHM.

The equation contains  $x$ . So it gives the displacement of **every element** of the string at any time. It tells us the **shape of the wave** at any instant. It also tells how this shape **changes** as the wave moves along. In short, it shows how the wave **progresses**.

WAVE ALONG +X

$$y(x, t) = a \sin(kx - \omega t + \phi)$$

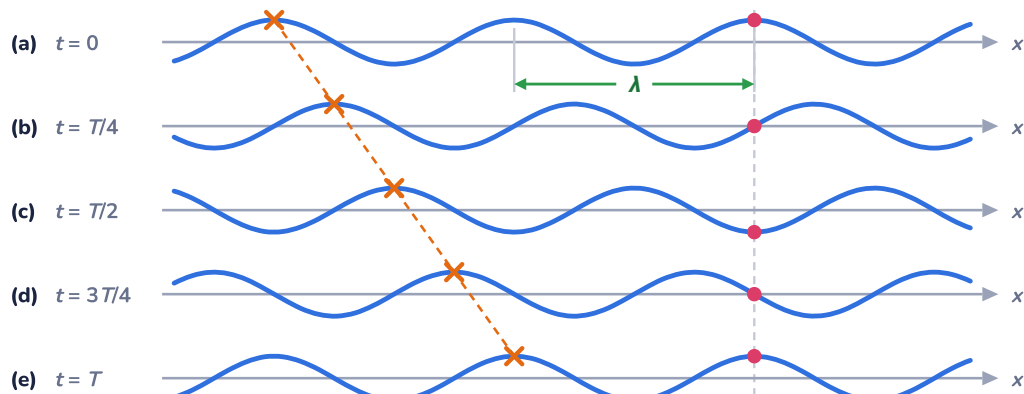
WAVE ALONG -X ... (II)

$$y(x, t) = a \sin(kx + \omega t + \phi)$$

### ● MEANING OF EACH SYMBOL

| SYMBOL                   | MEANING                                                                                         |
|--------------------------|-------------------------------------------------------------------------------------------------|
| $y(x, t)$                | Displacement of the vibrating element, as a function of <b>position</b> $x$ and <b>time</b> $t$ |
| $a$                      | <b>Amplitude</b> of the vibrating element (particle)                                            |
| $\omega$                 | <b>Angular frequency</b> of the wave                                                            |
| $k$                      | <b>Angular wave number</b>                                                                      |
| $x$                      | <b>Position</b> of the vibrating element                                                        |
| $(kx - \omega t + \phi)$ | <b>Phase</b> of the element at position $x$ and time $t$                                        |
| $\phi$                   | <b>Phase constant</b> : the phase at $x = 0, t = 0$                                             |

The figure plots Eq. (i) at **equal time steps** of a quarter period. A **crest** is a point of maximum positive displacement. A **trough** is a point of maximum negative displacement. The **cross (x)** marks one crest. It moves to the right as time passes. The **dot (•)** marks one element of the string. It only moves **up and down** at the same place.



A harmonic wave moving along  $+x$ , drawn at  $t = 0, \frac{T}{4}, \frac{T}{2}, \frac{3T}{4}, T$ . In one period the crest (x) moves one wavelength  $\lambda$

## 2.2 Amplitude and Phase

### DEFINITION

#### Amplitude

The **amplitude** of a wave is the **largest distance** that a particle of the medium moves away from its **equilibrium position** while the wave passes.

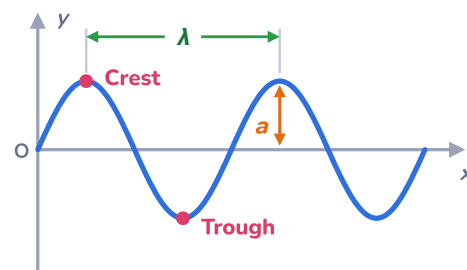
In Eq. (i),  $y(x, t)$  varies between

$+a$  and  $-a$

$y$  can be positive or negative, but  $a$  is

always positive

So  $a$  is the **largest displacement** of the particles of the medium from their **mean position**.



Crest, trough, amplitude  $a$  and wavelength  $\lambda$  on a wave

### DEFINITION

#### Phase

The **phase** of a wave is the quantity  $(kx - \omega t + \phi)$ . It is the **argument of the sine (or cosine) function** in the wave equation.

## 2.3 Wavelength and Angular Wave Number

### DEFINITION

#### Wavelength ( $\lambda$ )

The **wavelength** is the **smallest distance between two particles** in the **same phase**.

It is also the **distance the wave travels** while a particle completes **one oscillation**.

## DERIVATION

### Wavelength in Terms of $k$

Start from  $y(x, t) = a \sin(kx - \omega t + \phi)$ . Take  $\phi = 0$  and look at the wave at  $t = 0$ :

1

$$y(x, 0) = a \sin kx$$

2

The sine function repeats after every  $2\pi$ :

$$\sin kx = \sin(kx + 2n\pi) = \sin k \left( x + \frac{2n\pi}{k} \right)$$

3

So the displacement is the **same** at  $x$  and at  $x + \frac{2n\pi}{k}$ . Here  $n = 1, 2, 3$  and so on.

4

The **least** such distance is for  $n = 1$ :

$$\lambda = \left( x + \frac{2\pi}{k} \right) - x = \frac{2\pi}{k}$$

## WAVELENGTH

$$\lambda = \frac{2\pi}{k}$$

## ANGULAR WAVE NUMBER

$$k = \frac{2\pi}{\lambda} \text{ rad m}^{-1}$$

## DEFINITION

### Angular Wave Number ( $k$ )

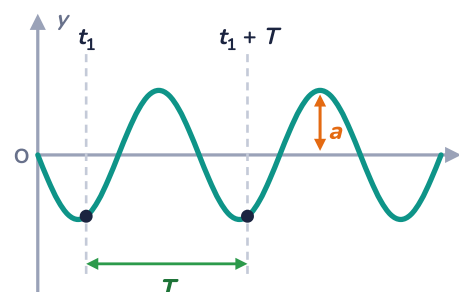
The **angular wave number**  $k$  of a wave is  $2\pi$  times the **number of waves** that fit in a **unit length**. It is also called the **propagation constant**. Its SI unit is  $\text{rad m}^{-1}$ .

## 2.4 Period, Angular Frequency and Frequency

Now watch **one element** of the string: the one at  $x = 0$ . Put  $\phi = 0$  and  $x = 0$  in  $y(x, t) = a \sin(kx - \omega t + \phi)$ :

$$y(0, t) = a \sin(-\omega t) = -a \sin \omega t$$

This graph is a **sine curve** in time. It shows the **displacement** of this element, its amplitude and its time period.



Element at  $x = 0$ :  $y = -a \sin \omega t$ . The motion repeats after every period  $T$

## • TIME PERIOD

### DEFINITION

The **time period** ( $T$ ) of a wave is the **time it takes to travel a distance of one wavelength**.

### DERIVATION

#### Angular Frequency

- 1 Let the period be  $T$ , counted from any time  $t_1$ . The displacement repeats after time  $T$ :

$$-a \sin \omega t_1 = -a \sin \omega(t_1 + T) = -a \sin(\omega t_1 + \omega T)$$

- 2 The sine function repeats after every  $2\pi$ . So:

$$\omega T = 2\pi$$

### ANGULAR FREQUENCY

$$\omega = \frac{2\pi}{T}$$

SI unit of  $\omega$ :  $\text{rad s}^{-1}$

## • FREQUENCY

### DEFINITION

The **frequency** ( $\nu$ ) of a wave is the **number of complete oscillations** it makes in **one second**. Its SI unit is the **hertz (Hz)**. It is related to the angular frequency  $\omega$  as shown below.

### FREQUENCY

$$\nu = \frac{1}{T} = \frac{\omega}{2\pi}$$

The **wavelength**  $\lambda$  and the **wave speed**  $v$  are related by:

$$\lambda = \frac{v}{\nu}$$

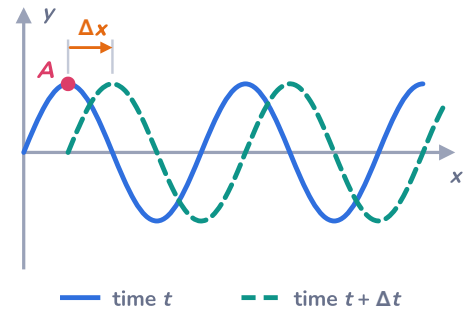
### NOTE

The **frequency of a wave never changes**, whatever interaction the wave goes through. It is fixed by the **source** alone.

## 2.5 Speed of a Travelling Wave

To find the speed of a wave, follow **any one point** on it. The **crest** is the easiest point to follow.

The figure shows the wave at two instants,  $t$  and  $t + \Delta t$ . In the time  $\Delta t$ , the whole pattern moves a distance  $\Delta x$ .



The wave moves to the right with velocity  $v$

**KEY**

Each point of the moving waveform, like point  $A$  on a crest, keeps its displacement  $y$ . The points of the string do not keep it. Only the points of the waveform do.

### DERIVATION

#### Speed of a Travelling Wave

- 1 Point  $A$  keeps its displacement as it moves. So its **phase stays constant**:

$$kx - \omega t = \text{constant}$$

- 2 Let  $x$  become  $x + \Delta x$  and  $t$  become  $t + \Delta t$ . The phase is still the same:

$$kx - \omega t = k(x + \Delta x) - \omega(t + \Delta t)$$

- 3 This gives  $k \Delta x = \omega \Delta t$ . So the wave speed is:

$$\frac{\Delta x}{\Delta t} = \frac{\omega}{k} = v$$

- 4 Put  $\omega = 2\pi\nu$  and  $\frac{2\pi}{k} = \lambda$ :

$$v = \frac{2\pi\nu}{k} = \nu \times \left(\frac{2\pi}{k}\right) = \nu\lambda$$

### SPEED OF A TRAVELLING WAVE

$$v = \nu\lambda = \frac{\lambda}{T}$$

**KEY**

This relation holds for all progressive waves. In one time period, the wave pattern moves a distance of one wavelength.

## 2.6 Speed of a Transverse Wave on a String

The speed of a **mechanical wave** depends on the **restoring force** in the medium. On a string, the **tension**  $T$  gives this force. We find the speed by the **method of dimensions**.

### DERIVATION

#### Speed on a String (Dimensional Method)

1 The speed can depend on two quantities. The **linear mass density** (mass per unit length) is  $\mu = [\text{ML}^{-1}]$ . The **tension** is  $T = [\text{MLT}^{-2}]$ .

2 Divide the dimensions of  $T$  by those of  $\mu$ :

$$\frac{[\text{MLT}^{-2}]}{[\text{ML}^{-1}]} = [\text{L}^2\text{T}^{-2}]$$

3 Speed has dimensions  $[\text{LT}^{-1}]$ . This is the square root of  $[\text{L}^2\text{T}^{-2}]$ . So:

$$v = c\sqrt{\frac{T}{\mu}}$$

4 Here  $c$  is a **dimensionless constant**. A full analysis shows that  $c = 1$ .

### SPEED OF A TRANSVERSE WAVE ON A STRING

$$v = \sqrt{\frac{T}{\mu}}$$

where,  $T$  = tension in the string,  $\mu$  = mass per unit length of the string

Depends on the **tension**  $T$  and the **linear mass density**  $\mu$

Yes

Depends on the **frequency** of the wave

No

The **source** of the wave fixes its frequency. The wavelength then follows from:

$$\lambda = \frac{v}{\nu}$$

## 2.7 Speed of Longitudinal Waves (Sound)

When a longitudinal wave passes, the particles of the medium move **forward and backward** along the direction of the wave. In a sound wave in air, they form **compressions** and **rarefactions**.

The restoring property of the medium is its **bulk modulus**  $B$ . Its SI unit is the **pascal (Pa)**. Here  $\Delta p$  = change in pressure,  $\Delta V$  = change in volume and  $V$  = initial volume:

$$B = -\frac{\Delta p}{\Delta V/V}$$

#### DERIVATION

#### Speed of Sound (Dimensional Method)

1 For a longitudinal wave, the **bulk modulus**  $B$  gives the restoring property. The **density**  $\rho$  gives the inertia. Their dimensions are  $B = [\text{ML}^{-1}\text{T}^{-2}]$  and  $\rho = [\text{ML}^{-3}]$ .

2 Divide the dimensions of  $B$  by those of  $\rho$ :

$$\frac{[\text{ML}^{-1}\text{T}^{-2}]}{[\text{ML}^{-3}]} = [\text{L}^2\text{T}^{-2}]$$

3 Match this with the dimensions of speed,  $[\text{LT}^{-1}]$ :

$$v = c\sqrt{\frac{B}{\rho}}$$

4 Experiments show that  $c = 1$ .

#### SPEED OF A LONGITUDINAL WAVE (SOUND) IN A MEDIUM

$$v = \sqrt{\frac{B}{\rho}}$$

#### Solid Bar (a Linear Medium)

For a **solid bar**, the right modulus is **Young's modulus**  $Y$ . The same dimensional method gives the speed of longitudinal waves in the bar:

$$v = \sqrt{\frac{Y}{\rho}}$$

where,  $Y$  = Young's modulus of the material of the bar,  $\rho$  = density of the material

#### • SPEED OF SOUND IN SOME MEDIA

| STATE | MEDIUM      | SPEED (M S <sup>-1</sup> ) |
|-------|-------------|----------------------------|
| Gases | Air (0 °C)  | 331                        |
|       | Air (20 °C) | 343                        |
|       | Helium      | 965                        |
|       | Hydrogen    | 1284                       |

| STATE   | MEDIUM            | SPEED (M S <sup>-1</sup> ) |
|---------|-------------------|----------------------------|
| Liquids | Water (0 °C)      | 1402                       |
|         | Water (20 °C)     | 1482                       |
|         | Seawater          | 1522                       |
| Solids  | Aluminium         | 6420                       |
|         | Copper            | 3560                       |
|         | Steel             | 5941                       |
|         | Granite           | 6000                       |
|         | Vulcanised rubber | 54                         |

## 2.8 Speed of Mechanical Waves

The speed of a mechanical wave depends on the **properties of the medium**. For one type of wave in a given medium, the speed **does not change** when these properties of the wave change:

× Amplitude

× Wavelength

× Frequency

e.g. **Infrasonic, audible** and **ultrasonic** sound waves in a given medium all have the speed given by:

### SPEED OF MECHANICAL WAVES

$$v = \sqrt{\frac{E}{\rho}}$$

where,  $E$  = the suitable **elastic constant** of the medium,  $\rho$  = **density** of the medium

**KEY** So this speed is decided by the medium alone.

## 2.9 Speed of Electromagnetic Waves

The medium also decides how fast **electromagnetic waves** travel. These waves are made of **electric and magnetic fields** that oscillate in **mutually perpendicular** directions.

Two properties of the medium decide their speed: the **permittivity**  $\epsilon$  and the **permeability**  $\mu$ .

### IN VACUUM (ALL EM WAVES)

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$\mu_0$  = permeability of vacuum,  $\epsilon_0$  = permittivity of vacuum

### IN A MATERIAL MEDIUM

$$v = \frac{1}{\sqrt{\mu \epsilon}}$$

$\mu$  = permeability of the medium,  $\epsilon$  = permittivity of the medium

## NOTE

In a material medium,  $\mu$  and  $\epsilon$  are **not exactly the same** for electromagnetic waves of **different frequencies**.

## 2.10 Newton's Formula for Speed of Sound

Newton assumed that the pressure and volume of a gas change **isothermally** (at constant temperature) when sound passes through it.

## DERIVATION

### Newton's Formula

- 1 For an ideal gas:

$$pV = Nk_B T$$

Here  $p$  = pressure,  $V$  = volume,  $N$  = number of molecules in the volume  $V$ ,  $k_B$  = Boltzmann constant and  $T$  = absolute temperature (in kelvin).

- 2 For an **isothermal** change,  $\Delta T = 0$ . Take small changes on both sides:

$$V \Delta p + p \Delta V = 0$$

- 3 Rearrange it into the form of the bulk modulus:

$$-\frac{\Delta p}{\Delta V/V} = p$$

- 4 The left side is the bulk modulus,  $B = -\frac{\Delta p}{\Delta V/V}$ . So the bulk modulus of the gas equals its pressure:

$$B = p$$

- 5 Put  $B = p$  in the formula for the speed of sound:

$$v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{p}{\rho}}$$

### NEWTON'S FORMULA

$$v = \sqrt{\frac{p}{\rho}}$$

where,  $p$  = pressure of the gas,  $\rho$  = density of the gas

## KEY

This is called Newton's formula for the speed of longitudinal waves (sound) in an ideal gas.

## 2.11 Laplace Correction

Newton's formula for air at **STP**

280 m s<sup>-1</sup> (about 15% too small)

**Experimental value** for air at STP

331 m s<sup>-1</sup>

**Laplace** found the mistake. When sound passes through a gas, its pressure and volume do **not** change isothermally. They change **adiabatically**.

### DERIVATION

#### Laplace Correction

1 For an **adiabatic** change:

$$pV^\gamma = \text{constant}$$

2 Take small changes in  $p$  and  $V$ :

$$p\gamma V^{\gamma-1}\Delta V + V^\gamma\Delta p = 0$$

3 So the **adiabatic bulk modulus** is:

$$B_{\text{ad}} = -\frac{\Delta p}{\Delta V/V} = \gamma p$$

Here  $\gamma$  is the ratio of the two specific heats,  $C_p/C_V$ .

### SPEED OF SOUND IN A GAS (LAPLACE)

$$v = \sqrt{\frac{\gamma p}{\rho}}$$

This change in Newton's formula is the **Laplace correction**. For air,  $\gamma = 7/5$ . The corrected formula then gives:

Speed of sound in air (Laplace's formula)

331.3 m s<sup>-1</sup>, matches experiment

### NOTE

#### Speed of Sound and the Temperature of the Gas

The gas equation for one mole is  $pV = RT$ . So  $p = \frac{RT}{V}$ . Put this in  $v = \sqrt{\gamma p/\rho}$ . Also,  $\rho \times V = M$ , the molar mass:

$$v = \sqrt{\frac{\gamma p}{\rho}} = \sqrt{\frac{\gamma RT}{\rho \times V}} = \sqrt{\frac{\gamma RT}{M}}$$

where,  $T$  = temperature of the gas,  $M$  = molar mass of the gas

## 03

## PART 3

## Superposition and Standing Waves

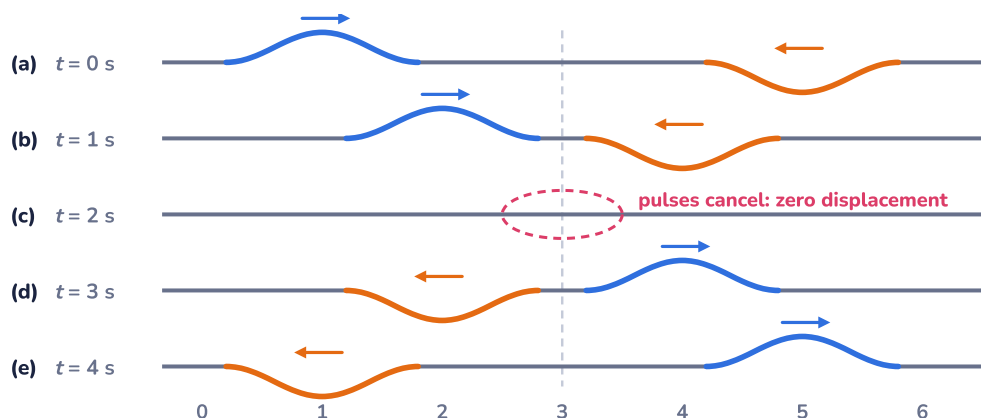
When two or more waves meet, their **displacements add up**. This one simple rule explains **interference**, **standing waves** and **beats**.

## 3.1 Principle of Superposition of Waves

## PRINCIPLE

The **principle of superposition** states that when two or more waves (or pulses) **overlap**, the **resultant displacement** at any point is the **algebraic sum** of the displacements due to each wave.

Two pulses travel along the **same stretched string** in **opposite directions**. They **meet**, **pass through each other** and then move on **independently**, without any change.



Two equal and opposite pulses on one string. At  $t = 2$  s they overlap exactly and cancel. Then each moves on unchanged

## KEY

In graph (c), the displacements due to the two pulses cancel exactly. The displacement is zero everywhere on the string.

## THREE IMPORTANT APPLICATIONS OF SUPERPOSITION

## 1 Standing waves

Two equal waves moving in **opposite directions**.

## 2 Beats

Two waves of **nearly equal frequencies**.

## 3 Interference

Waves meeting **in the same medium**.

## 3.2 Interference of Waves

## DEFINITION

**Interference** is the effect seen when **two or more waves** pass through the **same medium** at the same time. The medium takes a **shape** that comes from the **combined effect** of all the waves on its particles.

Let two waves produce displacements  $y_1(x, t)$  and  $y_2(x, t)$  in a medium. The **net displacement** is:

$$y(x, t) = y_1(x, t) + y_2(x, t)$$

When  $n$  waves overlap:

$$y = f_1(x - vt) + f_2(x - vt) + \dots + f_n(x - vt) = \sum_{i=1}^n f_i(x - vt)$$

Each function here stands for a **moving wave**. Superposition is the **basis of interference**. Take two harmonic waves with the **same angular frequency**  $\omega$ , the **same wave number**  $k = 2\pi/\lambda$  and the **same amplitude**  $a$ .

#### DERIVATION

#### Resultant of Two Interfering Waves

- 1 Let the two waves differ in phase by a **phase constant**  $\phi$ :

$$y_1(x, t) = a \sin(kx - \omega t)$$

$$y_2(x, t) = a \sin(kx - \omega t + \phi)$$

- 2 By the principle of superposition, the net displacement is:

$$y(x, t) = a \sin(kx - \omega t) + a \sin(kx - \omega t + \phi)$$

- 3 Use  $\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$ :

$$y = a \left[ 2 \sin \left\{ \frac{(kx - \omega t) + (kx - \omega t + \phi)}{2} \right\} \cos \frac{\phi}{2} \right]$$

#### RESULTANT WAVE

$$y(x, t) = 2a \cos \frac{\phi}{2} \sin \left( kx - \omega t + \frac{\phi}{2} \right)$$

The resultant is also a **harmonic wave** moving in the **positive  $x$ -direction**. It has the **same frequency and wavelength**. Its phase angle is  $\phi/2$ . Its **amplitude** is:

$$A(\phi) = 2a \cos \frac{\phi}{2}$$

#### I $\phi = 0$ (waves in phase)

$$y(x, t) = 2a \sin(kx - \omega t)$$

The amplitude is  $2a$ . This is the **maximum** amplitude.

#### II $\phi = \pi$ (out of phase by $180^\circ$ )

$$y(x, t) = 0$$

The displacement is **zero everywhere**, at **all times**.

### 3.3 Constructive and Destructive Interference

#### Constructive Interference

When the waves meet **in phase**, it is called **constructive interference**.

Phase difference

$$\phi = 2n\pi$$

Path difference

$$\Delta x = n\lambda$$

#### Destructive Interference

When the waves meet in **opposite phase**, it is called **destructive interference**.

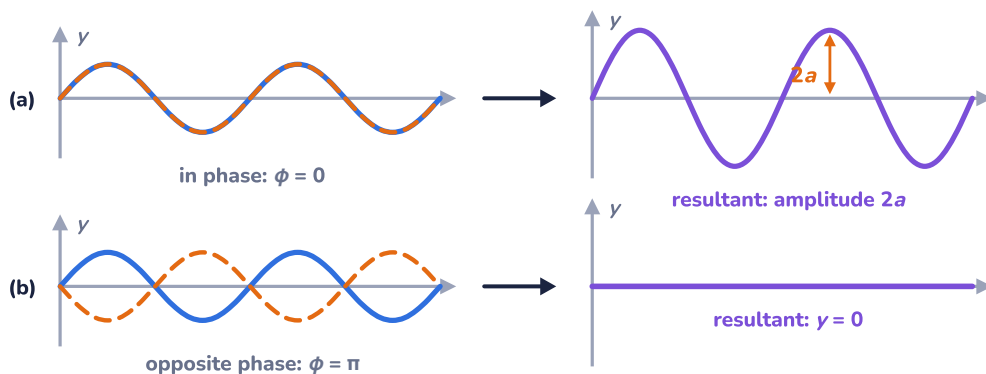
Phase difference

$$\phi = (2n + 1)\pi$$

Path difference

$$\Delta x = (2n + 1)\frac{\lambda}{2}$$

Here  $n = 0, 1, 2, 3$  and so on, and  $\lambda$  is the wavelength.



(a) Constructive interference: waves in phase add up. (b) Destructive interference: waves with phase difference  $\pi$  cancel

- ◆ **Constructive:** a crest of one wave meets a crest of another wave of the same frequency at a point. The resultant displacement is the **sum** of the two magnitudes. See Fig. (a).
- ◆ **Destructive:** a crest of one wave meets a trough of the other wave. The resultant displacement is the **difference** of the two magnitudes. See Fig. (b).
- ◆ Two sine waves of the **same amplitude and wavelength** may travel in the **same direction** on a stretched string. They combine into **one sine wave** that moves in that same direction.

#### ● INTENSITY IN INTERFERENCE

Intensity is proportional to the square of the amplitude,  $I \propto A^2$ . Take two sources with amplitudes  $A_1, A_2$  and intensities  $I_1, I_2$ :

#### RATIO OF MAXIMUM TO MINIMUM INTENSITY

$$\frac{I_{\max}}{I_{\min}} = \left( \frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}} \right)^2$$

When  $I_1 = I_2 = I_0$ , the resultant intensity is:

$$I_{\text{resultant}} = 4I_0 \cos^2(\phi/2)$$

where,  $\phi$  = phase difference between the two waves

## NOTE

These rules also hold for **sound waves** and **light waves**. But the sources must be **coherent**. This means their **phase difference must not change** with time.

### 3.4 Reflection of Waves

A pulse or wave that meets a **rigid boundary** is **reflected**. An **echo** is an example. If the boundary is **not fully rigid**, some of the wave comes back (**reflected**). The rest goes on into the second medium (**transmitted**).

A wave may fall **obliquely** on the boundary between two media. Then the transmitted wave is called the **refracted wave**.

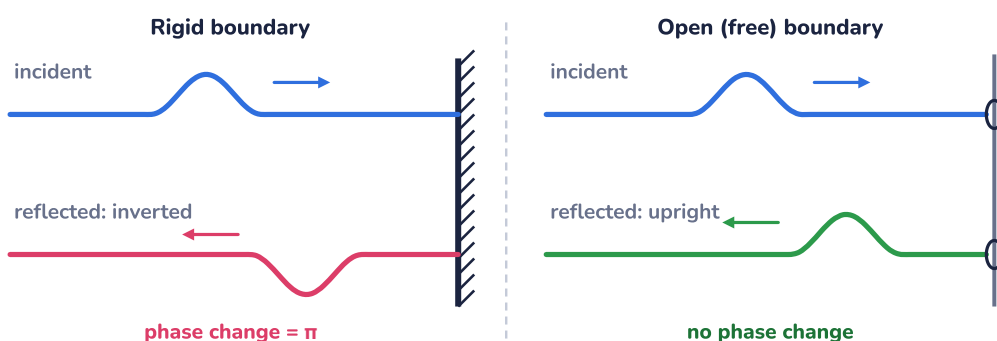
**Incident** and **refracted** waves obey

Snell's law of refraction

**Incident** and **reflected** waves obey

The usual laws of reflection

Look at a pulse moving towards a **rigid wall** that absorbs **no energy**. The point at the wall cannot move, so its displacement must stay **zero**. By the principle of superposition, the reflected wave must have the **same amplitude** and a **phase difference of  $\pi$** .



Reflection of a pulse. At a rigid boundary it comes back inverted (phase change  $\pi$ ). At a free boundary it comes back upright

Let the **incident wave** be:

$$y(x, t) = a \sin(kx - \omega t)$$

#### i At a Rigid Boundary

The reflected wave has a **phase change of  $\pi$** , so it is inverted. At the boundary:

$$y_r(x, t) = a \sin(kx - \omega t + \pi)$$

$$= -a \sin(kx - \omega t)$$

Net displacement at the boundary:  $y' = y + y_r = 0$  at **all times**.

### ii At an Open Boundary

The boundary point is **completely free to move**. The reflected pulse has the **same phase and amplitude**:

$$y_r(x, t) = a \sin(kx - \omega t + 0)$$

$$= a \sin(kx - \omega t)$$

Net maximum displacement at the boundary = **twice the amplitude**.

## 3.5 Standing Waves

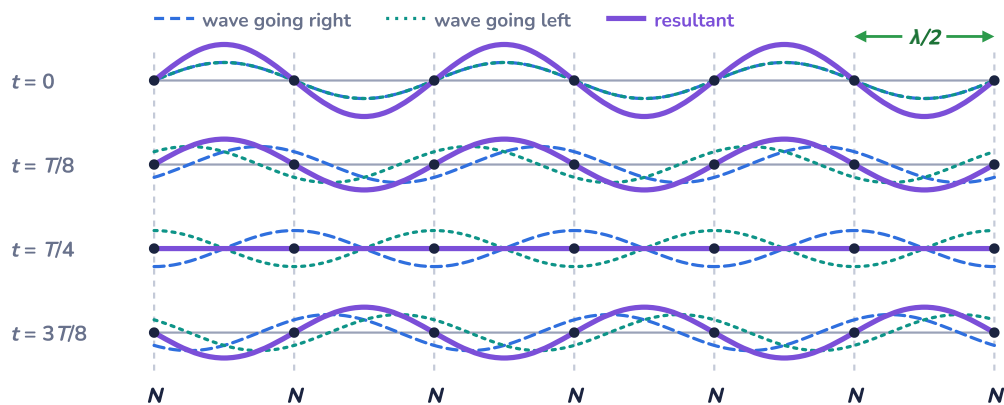
### DEFINITION

**Standing waves**, or **stationary waves**, are formed when two **progressive waves of the same type** overlap while moving along the **same straight line in opposite directions**. The two waves have the **same amplitude**, the **same frequency** (time period and wavelength) and the **same speed**.

"Same type" means both are transverse, or both are longitudinal.

**KEY** A standing wave does not move in any direction. It carries no energy through the medium.

Think of a string whose two ends are **both fixed**. Or think of air in a pipe that is **closed at both ends**. The wave reflects at both boundaries. The **incident and reflected waves interfere**. So a **steady wave pattern** sets up on the string.



Two harmonic waves moving in opposite directions give a stationary wave. Its nodes N stay at fixed places,  $\lambda/2$  apart

### DERIVATION

#### Equation of a Stationary Wave

1 Waves along the **+x** and the **-x** directions:

$$y_1(x, t) = a \sin(kx - \omega t) \quad y_2(x, t) = a \sin(kx + \omega t)$$

2 The resultant wave:

$$y(x, t) = y_1(x, t) + y_2(x, t) = a [\sin(kx - \omega t) + \sin(kx + \omega t)]$$

3 Use  $\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$ . This gives the result below.

#### STATIONARY WAVE

$$y(x, t) = 2a \sin kx \cos \omega t$$

The **amplitude** of the stationary wave is  $2a \sin kx$ .

- ◆ The amplitude **changes from point to point**.
- ◆ Every element oscillates with the **same angular frequency**  $\omega$  (same time period). There is **no phase difference** between the oscillations of different elements.
- ◆ The pattern stays in place. The amplitude is **fixed at a given point**, but **different at different points**.

### 3.6 Nodes, Antinodes and Normal Modes

#### Node (N)

A **node** is a point of a standing wave where the amplitude is **zero**. There is **no motion** at a node.

#### Antinode (A)

An **antinode** is a point of a standing wave where the amplitude is the **largest**.

#### CONCEPT

#### Normal Modes

A system with standing waves **cannot oscillate at any frequency it likes**. It has a fixed set of **natural frequencies**. These are called its **normal modes of oscillation**.

We now find them for a **stretched string with both ends fixed**.

#### DERIVATION

#### Position of Nodes

1 At a node, the amplitude  $2a \sin kx$  is zero:

$$\sin kx = 0 \Rightarrow kx = n\pi; n = 0, 1, 2, 3, 4, \dots$$

2 Put  $k = \frac{2\pi}{\lambda}$ :

$$x = \frac{n\pi}{k} = \frac{n\pi}{2\pi/\lambda}$$

#### POSITION OF NODES

$$x = \frac{n\lambda}{2}; n = 0, 1, 2, 3, \dots$$

First node at  $x = 0$ , second node at

$$x = \lambda/2$$

Distance between two **successive nodes**

$$\lambda/2$$

#### DERIVATION

##### Position of Antinodes

- 1 At an antinode, the amplitude has its largest value,  $2a$ . This happens when  $|\sin kx| = 1$ :

$$kx = \left(n + \frac{1}{2}\right) \pi; n = 0, 1, 2, 3, \dots$$

- 2 Put  $k = \frac{2\pi}{\lambda}$  to get the positions below.

#### POSITION OF ANTINODES

$$x = (2n + 1) \frac{\lambda}{4}; n = 0, 1, 2, \dots$$

At these points of the string, the amplitude is **maximum**.

First antinode at  $x = \lambda/4$ , second at

$$x = 3\lambda/4$$

Distance between two **neighbouring antinodes**

$$\lambda/2$$

**Node to node =  $\lambda/2$  · Antinode to antinode =  $\lambda/2$**

### 3.7 String Fixed at Both Ends

Take a stretched string of **length  $L$** , fixed at **both ends**. Its **boundary conditions** are:

$x = 0$  one fixed end

$x = L$  other fixed end

Both ends must be nodes. So the length must hold a **whole number of half-wavelengths**:

$$L = \frac{n\lambda}{2}; n = 1, 2, 3, \dots \Rightarrow \text{possible wavelengths } \lambda = \frac{2L}{n}; n = 1, 2, 3, \dots$$

## NATURAL FREQUENCIES OF THE STRING

$$\nu = \frac{nv}{2L} \quad \text{for } n = 1, 2, 3 \dots$$

where,  $v$  = speed of the wave on the string,  $L$  = length of the string

### ● MODES OF VIBRATION

#### DEFINITION

The **mode of vibration** of a string is the **way it vibrates** to form a standing wave.

#### First Mode ( $n = 1$ )

The string vibrates in **one loop**. This is the **lowest natural frequency**. It is called the **fundamental mode** or **first harmonic**.

$$L = \frac{\lambda_1}{2}, \quad \nu = \frac{v}{2L}$$

#### Second Mode ( $n = 2$ )

The string vibrates in **two loops**. It is called the **second harmonic** or **first overtone**.

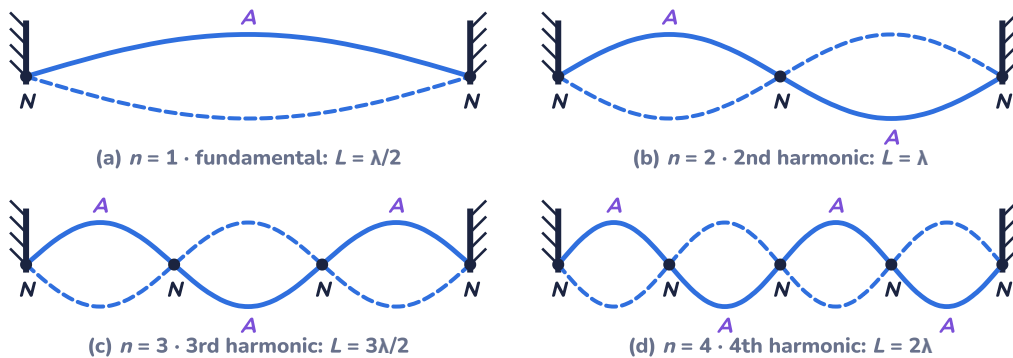
$$L = \lambda_2, \quad \nu_2 = \frac{2v}{2L} = 2\nu_1$$

#### Third Mode ( $n = 3$ )

The string vibrates in **three loops**. It is called the **third harmonic**. Higher modes follow in the same way.

$$\nu = \frac{3v}{2L} = 3\nu_1$$

The figure shows the **first four harmonics**. A real string vibrates in a **mixture of several modes** at the same time. Some modes are **strongly excited**, others only weakly.



First four harmonics of a string fixed at both ends. The fixed ends are always nodes

#### NOTE

Musical instruments like the **sitar** and the **violin** work on this principle. **Where the string is plucked or bowed** decides which modes are stronger than the others.

### 3.8 String Fixed at One End

A standing wave can also form on a string **fixed at one end**. Its other end can **move freely** sideways. Then:

The **free end** is always

an antinode (A)

## DERIVATION

## Frequencies of a String Fixed at One End

- 1 Take the standing wave with a node at the fixed end,  $x = 0$ :

$$y(x, t) = 2a \sin kx \cos \omega t$$

- 2 For an **antinode at**  $x = L$ , where  $n = 0, 1, 2$  and so on:

$$\sin kL = \pm 1 \Rightarrow kL = (2n + 1) \frac{\pi}{2}$$

- 3 Put  $k = \frac{2\pi}{\lambda}$ :

$$\frac{2\pi}{\lambda} \times L = (2n + 1) \frac{\pi}{2} \Rightarrow \frac{2L}{\lambda} = (2n + 1) \times \frac{1}{2}$$

- 4 Use  $\lambda = \frac{v}{\nu}$ :

$$\frac{2L\nu}{v} = (2n + 1) \frac{1}{2}$$

## FREQUENCY OF VIBRATION

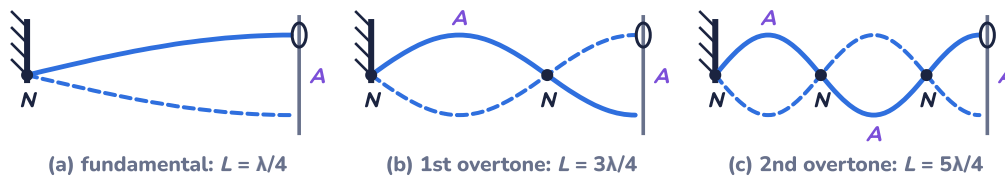
$$\nu = \frac{(2n + 1)}{4L} v$$

where,  $n = 0, 1, 2$  and so on,  $\nu$  = frequency,  $v$  = speed of the wave

These are the **normal frequencies** of the string. The **fundamental frequency** is for  $n = 0$ . The other frequencies are:

| FREQUENCY                        | NAME                                     |
|----------------------------------|------------------------------------------|
| $\nu_0 = \frac{v}{4L}$           | <b>First harmonic</b> (fundamental)      |
| $\nu_1 = \frac{3v}{4L} = 3\nu_0$ | <b>Third harmonic</b> (first overtone)   |
| $\nu_2 = \frac{5v}{4L} = 5\nu_0$ | <b>Fifth harmonic</b> (second overtone)  |
| $\nu_3 = \frac{7v}{4L} = 7\nu_0$ | <b>Seventh harmonic</b> (third overtone) |

**$\nu_0 : \nu_1 : \nu_2 : \nu_3 = 1 : 3 : 5 : 7 \Rightarrow$  Only odd harmonics are present**



(a) Fundamental, (b) first overtone and (c) second overtone. Node N at the fixed end, antinode A at the free end

#### LAW

### Laws of Transverse Vibration of a String

Given string, fixed tension  $T$ : frequency is **inversely proportional to the vibrating length  $L$**

$$\nu \propto \frac{1}{L}$$

Uniform string of fixed length: frequency is **proportional to the square root of the tension  $T$**

$$\nu \propto \sqrt{T}$$

Fixed length and tension: frequency is **inversely proportional to the square root of the mass per unit length**

$$\nu \propto \frac{1}{\sqrt{\mu}}$$

A **sonometer** measures the **fundamental frequency** of a vibrating string. It is also used to **verify** these laws.

## 04 PART 4 Organ Pipes and Beats

Air inside a pipe can also form **standing waves**. This part also explains **beats**, which come from two waves of nearly equal frequency.

### 4.1 Vibrations of Air Columns

Air vibrating inside an **organ pipe** is a familiar case of **stationary waves**. The pipe is a tube shaped like a **cylinder**. There are two kinds:

#### i Closed Organ Pipe

**Closed at one end** and open at the other end.

#### ii Open Organ Pipe

**Open at both ends.**

Make the air at the open end of the pipe vibrate. A **longitudinal wave** travels along the pipe to its **far end**. There it is **reflected back**. The **incident** and **reflected** waves overlap. So **stationary waves form in the pipe**.

**Air vibrates at the open end → Longitudinal wave travels → Reflected at the far end → Stationary waves**

## 4.2 Closed Organ Pipe

Now find the **normal modes** of an air column that is **closed at one end** and **open at the other**. This is a closed organ pipe. A **glass tube partly filled with water** is a simple example.

The **closed end** (at the water surface) is a **node**

Pressure changes the most

The **open end** is an **antinode**

Pressure changes the least

### DERIVATION

#### Frequencies of a Closed Organ Pipe

- Take the end touching the water as  $x = 0$  (a node). The other end,  $x = L$ , is an antinode. So:

$$|\sin kx| = 1 \Rightarrow |\sin kL| = 1$$

- The sine has magnitude 1 at odd multiples of  $\pi/2$ :

$$kL = \left(n + \frac{1}{2}\right) \pi$$

- Put  $k = \frac{2\pi}{\lambda}$ :

$$L = (2n + 1) \frac{\lambda}{4}; n = 0, 1, 2, 3, \dots$$

### FREQUENCY OF A CLOSED ORGAN PIPE

$$\nu = (2n + 1) \frac{v}{4L}; n = 0, 1, 2, 3, \dots$$

where,  $v$  = speed of sound in air,  $L$  = length of the air column

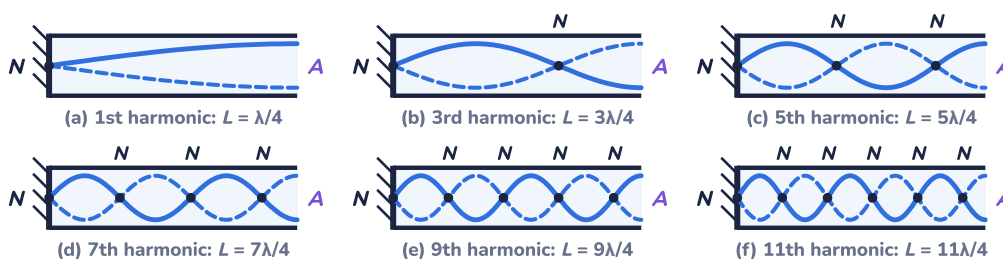
**Fundamental frequency** ( $n = 0$ )

$$\nu = \frac{v}{4L}$$

Higher frequencies: the **odd harmonics**

$$\frac{3v}{4L}, \frac{5v}{4L}, \text{etc.}$$

**KEY** A pipe closed at one end gives only odd harmonics.



Odd harmonics of an air column closed at the left end and open at the right end. (a) is the fundamental. Dots mark the nodes

### NOTE

- ◆ A system is in **resonance** when the outside (driving) frequency is close to one of its **natural frequencies**.
- ◆ Take a **circular membrane** clamped tightly all around its edge. Its normal modes follow from one boundary condition: **no point on the edge vibrates**.
- ◆ Standing waves in a **closed organ pipe** are like those on a **string fixed at one end**.

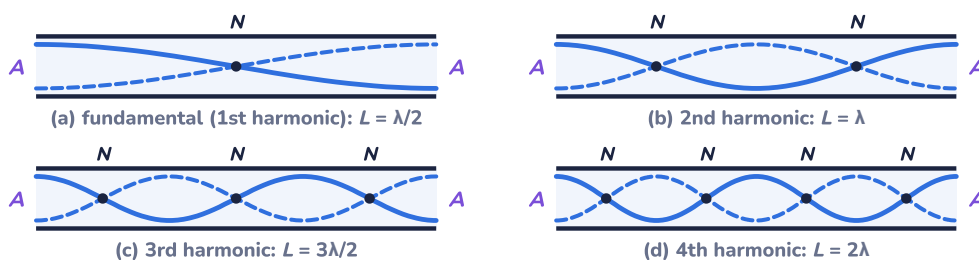
## 4.3 Open Organ Pipe

In a pipe **open at both ends**, **each end is an antinode**. Such an air column gives **all the harmonics**. Its frequency formula is the **same as for a string fixed at both ends**.

### FREQUENCY OF AN OPEN ORGAN PIPE

$$\nu = \frac{nv}{2L} \quad \text{for } n = 1, 2, 3, \dots$$

**KEY** Both ends are antinodes. Nodes and antinodes alternate between them.



First four harmonics of an open organ pipe: antinode A at each open end, nodes N in between

| $n$ | FREQUENCY                        | NAME                                    |
|-----|----------------------------------|-----------------------------------------|
| 1   | $\nu_1 = \frac{v}{2L}$           | <b>Fundamental</b> or first harmonic    |
| 2   | $\nu_2 = \frac{v}{L} = 2\nu_1$   | <b>Second harmonic</b> (first overtone) |
| 3   | $\nu_3 = \frac{3v}{2L} = 3\nu_1$ | <b>Third harmonic</b> (second overtone) |
| 4   | $\nu_4 = \frac{4v}{2L} = 4\nu_1$ | <b>Fourth harmonic</b> (third overtone) |

$\nu_1 : \nu_2 : \nu_3 = 1 : 2 : 3 \Rightarrow$  An open pipe gives all harmonics

**NOTE**

- ◆ An **open pipe** gives a **shriller** sound (higher pitch) than a **closed pipe of the same length**.
- ◆ The **quality of sound** from an open pipe is **better** than from a closed pipe.
- ◆ Standing waves in an **open organ pipe** are like those on a **string fixed at both ends**.

**4.4 Beats**

Beats come from the **interference** of two waves with **nearly equal frequencies**.

**DEFINITION**

**Beats** are the **periodic rise and fall in the intensity of sound**. They are heard when two sound waves of **nearly equal frequencies and amplitudes**, moving in the **same direction**, overlap.

Sound gets louder once and softer once

One beat

Beats heard in **one second**

Beat frequency

**REMEMBER****The 10 Hz Limit**

The two frequencies should **not differ by more than 10 Hz**. For a bigger difference, the ear **cannot tell the rises and falls apart**. This is due to the **persistence of hearing**.

**GOOD TO KNOW****Tuning Instruments**

Musicians use beats to **tune their instruments** with each other. They keep tuning until their trained ears **hear no beats**. The frequencies of all the instruments in the band then **match**.

**4.5 Analytical Treatment of Beats**

Take two harmonic sound waves with **nearly equal angular frequencies**  $\omega_1$  and  $\omega_2$ . Look only at the point  $x = 0$ . These are sound waves, so  $y_1$  and  $y_2$  stand for **longitudinal displacements**.

**DERIVATION****Beat Frequency**

- 1 The two waves at  $x = 0$ :

$$y_1 = a \cos \omega_1 t \quad \text{and} \quad y_2 = a \cos \omega_2 t$$

2 By the principle of superposition, the resultant displacement is:

$$y = y_1 + y_2 = a [\cos \omega_1 t + \cos \omega_2 t]$$

3 Use  $\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$ . Write  $\omega_b = \frac{\omega_1 - \omega_2}{2}$  and  $\omega_a = \frac{\omega_1 + \omega_2}{2}$ :

$$y = 2a \cos \frac{(\omega_1 - \omega_2)t}{2} \cos \frac{(\omega_1 + \omega_2)t}{2}$$

$$\Rightarrow y = 2a \cos \omega_b t \cos \omega_a t$$

4 Assume  $|\omega_1 - \omega_2| \ll \omega_1, \omega_2$ . Then  $\omega_a \gg \omega_b$ . So the resultant oscillates with the **average angular frequency**  $\omega_a$ . Its amplitude is **maximum** when:

$$|\cos \omega_b t| = 1$$

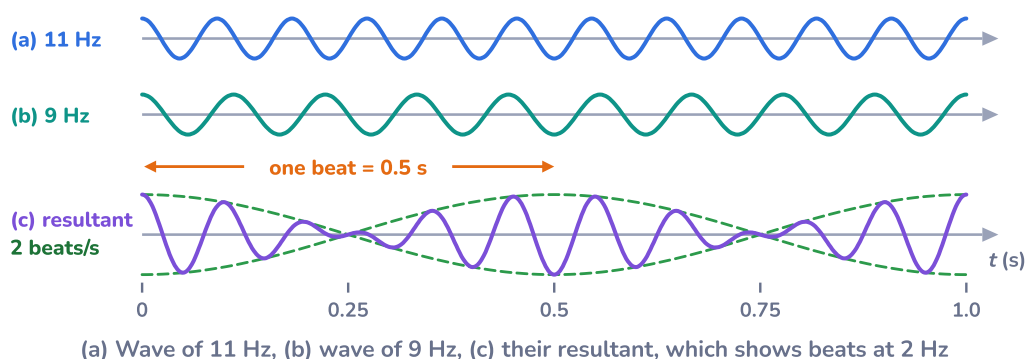
5 So the resultant wave **waxes and wanes** (grows louder and softer) with angular frequency:

$$2\omega_b = \omega_1 - \omega_2$$

#### BEAT FREQUENCY

$$\nu_{\text{beat}} = \nu_1 - \nu_2$$

The figure shows beats from two harmonic waves of **11 Hz** and **9 Hz**. The amplitude of the resultant wave rises and falls **twice every second**. So the beat frequency is **2 Hz**.



#### GOOD TO KNOW

##### Musical Pillars

Some temple pillars of rock give **basic musical notes** when tapped. They are of **three types**:

- ◆ **Shruti pillars** produce the basic notes, the '**swaras**'.
- ◆ **Gana Thoongal** produce the basic tunes that make up the '**ragas**'.
- ◆ **Laya Thoongal** produce '**taal**' (**beats**) when tapped.

| TERM                                   | KEY POINTS                                                                                                                                                                                                                                                 |
|----------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Wave motion</b>                     | A disturbance that travels through a medium because its particles oscillate again and again about their mean positions. Each particle hands the disturbance to the next. Three types: <b>mechanical</b> , <b>electromagnetic</b> and <b>matter waves</b> . |
| <b>Transverse wave</b>                 | Particles oscillate <b>perpendicular</b> to the wave. Needs a medium with a <b>shear modulus</b> , e.g. a solid.                                                                                                                                           |
| <b>Longitudinal wave</b>               | Particles oscillate <b>along</b> the wave. Needs a <b>bulk modulus</b> , so it travels in all media: solids, liquids and gases.                                                                                                                            |
| <b>Progressive wave</b>                | A wave that moves from one point of the medium to another. $y(x, t) = a \sin(kx - \omega t + \phi)$ along $+x$ , and $y(x, t) = a \sin(kx + \omega t + \phi)$ along $-x$ .                                                                                 |
| <b>Amplitude</b>                       | Maximum displacement from the mean position.                                                                                                                                                                                                               |
| <b>Time period <math>T</math></b>      | Time for one complete oscillation.                                                                                                                                                                                                                         |
| <b>Frequency <math>\nu</math></b>      | Number of waves produced per second.                                                                                                                                                                                                                       |
| <b>Wavelength <math>\lambda</math></b> | Distance between the two nearest particles oscillating in the same phase.                                                                                                                                                                                  |
| <b>Basic relations</b>                 | $\lambda = \frac{2\pi}{k}, \omega = \frac{2\pi}{T}, \nu = \frac{1}{T}, v = \nu\lambda = \frac{\lambda}{T}$                                                                                                                                                 |
| <b>Wave speeds</b>                     | String: $v = \sqrt{\frac{T}{\mu}}$ · Mechanical: $v = \sqrt{\frac{E}{\rho}}$<br>Solid: $v = \sqrt{\frac{Y}{\rho}}$ · Liquid: $v = \sqrt{\frac{B}{\rho}}$<br>Gas (Newton): $v = \sqrt{\frac{P}{\rho}}$ · Laplace: $v = \sqrt{\frac{\gamma P}{\rho}}$        |
| <b>Speed of sound</b>                  | $v_{\text{solid}} > v_{\text{liquid}} > v_{\text{gas}}$                                                                                                                                                                                                    |
| <b>Superposition</b>                   | $y'(x, t) = y_1(x, t) + y_2(x, t)$                                                                                                                                                                                                                         |
| <b>Interference</b>                    | $y = 2a \cos \frac{\phi}{2} \sin \left( kx - \omega t + \frac{\phi}{2} \right)$<br>In phase ( $\phi = 0$ ): $y = 2a \sin(kx - \omega t)$ · Out of phase ( $\phi = \pi$ ): $y = 0$                                                                          |
| <b>Constructive / destructive</b>      | Constructive: $\phi = 2n\pi, \Delta x = n\lambda$ . Destructive: $\phi = (2n + 1)\pi, \Delta x = (2n + 1)\frac{\lambda}{2}$ .                                                                                                                              |
| <b>Phase and path difference</b>       | $\phi = \frac{2\pi}{\lambda} \times x \Rightarrow x = \frac{\phi\lambda}{2\pi}$                                                                                                                                                                            |
| <b>Organ pipes</b>                     | Closed pipe: $\nu = (2n + 1) \frac{v}{4L}$ · Open pipe: $\nu = \frac{nv}{2L}$                                                                                                                                                                              |
| <b>Beats</b>                           | $\nu_{\text{beat}} = \nu_1 - \nu_2$                                                                                                                                                                                                                        |