

CHAPTER

05

Work, Energy and Power

Work • Kinetic Energy & Work-Energy Theorem • Potential Energy & Conservation of Energy • Power & Collisions

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READING KEY

DEFINITION

NOTE

REMEMBER

DERIVATION

STRATEGY

Formulas sit on graph paper • coloured words are the key terms

01 PART 1 Work

In daily life, any **physical or mental effort** is called work. In physics, the word **work** has an exact meaning. Work is done only when a **force moves a body**. **Energy** is the ability of a body to do work. **Power** tells us how fast work is done. These three ideas are linked by the **work-energy theorem**, the **conservation of energy** and **collisions**.

1.1 Work

DEFINITION

Work is done by a force when it **moves a body** some distance **along its own direction**.

So a force does work only when **two conditions** are met:

i Condition

A **force acts** on the body.

ii Condition

The **point where the force acts** moves **along the force**.

EVERYDAY EXAMPLES OF WORK

An **engine pulls a train** on the rails

Work is done ✓

A **man climbs** a hill

Work is done ✓

A **horse pulls a cart** along a road

Work is done ✓

1.2 Work Done by a Constant Force

DEFINITION

The **work done by a constant force** = (component of the force along the displacement) × (magnitude of the displacement).

Let a **constant force** $\vec{F}$ act on a body. The body moves through a displacement $\vec{s}$ in the **same direction as the force**. Then the work done is simply the force multiplied by the displacement.

FORCE ALONG THE DISPLACEMENT

$$W = \vec{F} \cdot \vec{s} = Fs$$

where, F = magnitude of the constant force, s = magnitude of the displacement



Force F moves the block through a displacement s along the force

NOTE

Work has **magnitude only**. It has no direction. So work is a **scalar quantity**. We get it from the **dot (scalar) product** of $\vec{F}$ and $\vec{s}$.

1.3 Work Done by an Inclined Force

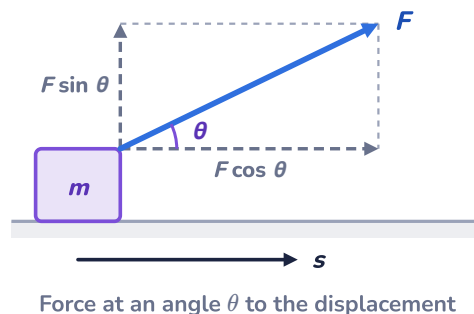
Often the force acts at an **angle θ** to the displacement $\vec{s}$. Then we **resolve $\vec{F}$** into **two rectangular components**.

Along $\vec{s}$: $F_x = F \cos \theta$

This part points **along the displacement**. So **all the work** is done by it.

Perpendicular to $\vec{s}$: $F_y = F \sin \theta$

This part is **at 90° to the displacement**. So its work is **zero**.



Only $F \cos \theta$ does work. So the work done is $F \cos \theta$ multiplied by s :

WORK DONE BY AN INCLINED FORCE

$$W = \vec{F} \cdot \vec{s} = F s \cos \theta$$

W is the **dot (scalar) product** of $\vec{F}$ and $\vec{s}$, so work is a **scalar**.

$\theta = 0^\circ$ • **Same Direction**

$$W = F s \cos 0^\circ = F s$$

The work done is **maximum**.

$\theta = 90^\circ$ • **Perpendicular**

$$W = F s \cos 90^\circ = 0$$

The force does **zero work**.

$\theta = 180^\circ$ • **Opposite**

$$W = F s \cos 180^\circ = -F s$$

The work done is **negative**.

NOTE

If the displacement is **perpendicular** to the force, the work done is **zero**. The type of motion does not matter. The body may move with constant velocity, constant acceleration, and so on. **e.g.** The **centripetal force** in circular motion does no work.

1.4 Dimensions and Units of Work

DIMENSIONAL FORMULA OF WORK

$$W = \text{force} \times \text{distance} = [\text{MLT}^{-2}] \times [\text{L}] = [\text{ML}^2\text{T}^{-2}]$$

Work has **two types of units**: **absolute units** and **gravitational units**.

1 Absolute Units

One absolute unit of work is done when a **unit force** moves a body through a **unit distance** along the force.

- ◆ **Joule (SI)**: $1 \text{ J} = 1 \text{ N} \times 1 \text{ m} = 1 \text{ N-m}$
- ◆ **Erg (CGS)**: $1 \text{ erg} = 1 \text{ dyne} \times 1 \text{ cm} = 1 \text{ dyne-cm}$

2 Gravitational Units

One gravitational unit of work is done when a **gravitational unit of force** moves a body through a **unit distance** along the force.

- ◆ **kg-m (SI)**: $1 \text{ kg-m} = 9.8 \text{ J}$
- ◆ **g-cm (CGS)**: $1 \text{ g-cm} = 981 \text{ erg}$

$$1 \text{ J} = 10^7 \text{ erg} \cdot 1 \text{ kg-m} = 9.8 \text{ J} \cdot 1 \text{ g-cm} = 981 \text{ erg}$$

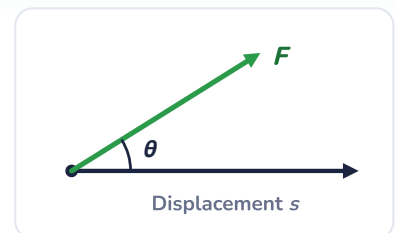
1.5 Positive, Negative and Zero Work

We know that $W = Fs \cos \theta$. So the **sign of the work** depends on the **angle θ** between the force and the displacement. The work done may be **positive**, **negative** or **zero**.

1 Positive Work • $0 \leq \theta < 90^\circ$

The force has a part **along the displacement**. So $\cos \theta$ is **positive**, and the work is positive.

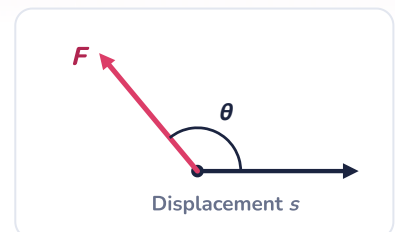
- ◆ **Horse pulling a cart**: the force and the motion are in the same direction.
- ◆ **Lifting a load**: the lifting force and the displacement both point up.
- ◆ **Stretching a spring**: the stretching force acts along the displacement.



2 Negative Work • $90^\circ < \theta \leq 180^\circ$

The force has a part **opposite to the displacement**. So $\cos \theta$ is **negative**, and the work is negative.

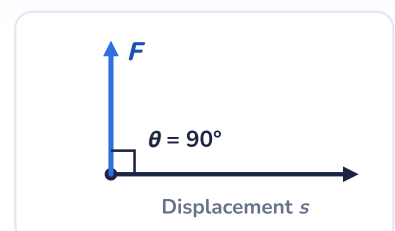
- ◆ **Brakes on a moving vehicle**: the braking force acts against the motion.
- ◆ **Friction** when a body is pulled along a rough floor.
- ◆ **Gravity** when a body is raised upward.



3 Zero Work • $\theta = 90^\circ$

The body moves **at right angles to the force**. Since $\cos 90^\circ = 0$, the work is $W = Fs \cos 90^\circ = 0$.

- ◆ A **coolie** walks on level ground with a load on his head.
- ◆ The **centripetal force** when a body moves in a circle at constant speed.
- ◆ The **weight** of a body sliding on a smooth (frictionless) level floor.



NATURE OF WORK	ANGLE θ	SIGN OF $\cos \theta$	WORK DONE
Positive	$0 \leq \theta < 90^\circ$	Positive (+)	$W = Fs \cos \theta$ is positive
Negative	$90^\circ < \theta \leq 180^\circ$	Negative (-)	$W = Fs \cos \theta$ is negative
Zero	$\theta = 90^\circ$	Zero (0)	$W = 0$

1.6 Work Done by a Variable Force

Sometimes the force is **not constant**. It changes with position, say along the x -axis. Then $W = Fs$ cannot be used. So we break the path into **very small displacements** Δx . Over each small step, the force is almost **constant**. Then we add the work of all the steps.

DERIVATION

- Over a tiny step Δx , the force $F(x)$ hardly changes. The work in this step equals the **area of the thin strip** $abcd$.

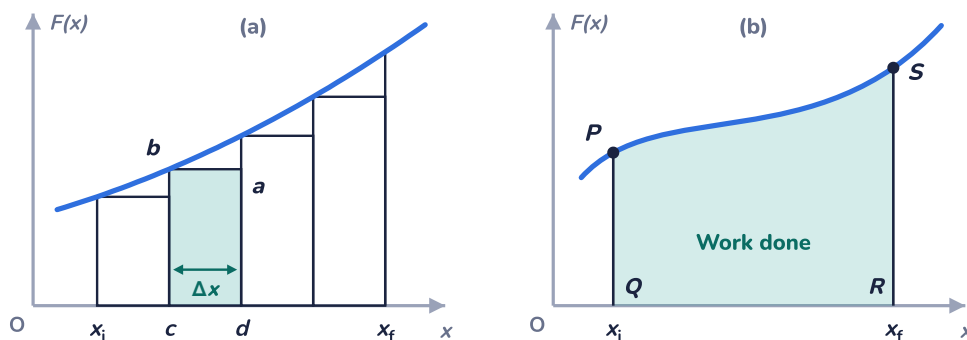
$$\Delta W = F(x) \Delta x = \text{area of strip}$$

- Add the areas of **all the strips** from x_i to x_f . This gives the total work approximately.

$$W \approx \sum_{x_i}^{x_f} F(x) \Delta x$$

- Now let $\Delta x \rightarrow 0$. The sum becomes an **integral**. It gives the exact area $PQRS$ under the curve.

$$W = \lim_{\Delta x \rightarrow 0} \sum F(x) \Delta x = \int_{x_i}^{x_f} F(x) dx$$



(a) Work $\approx$ total area of the thin strips · (b) Exact work = shaded area $PQRS$

WORK DONE BY A VARIABLE FORCE

$$W_{x_i \rightarrow x_f} = \int_{x_i}^{x_f} F dx = \int_{x_i}^{x_f} (F \cos \theta) dx = \text{Area under the } F\text{-}x \text{ curve}$$

Force in Three Dimensions

When both the force and the displacement change in 3-D, split each one into **rectangular components**:

$$\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}, \quad d\vec{s} = dx \hat{i} + dy \hat{j} + dz \hat{k}$$

Then the work done is:

$$W = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy + \int_{z_i}^{z_f} F_z dz$$

NOTE

A body in **equilibrium** (static or dynamic) has **zero resultant force**. So **no net work** is done on it. The total work is **zero**.

1.7 Conservative Forces

DEFINITION

A **conservative force** is a force whose work is fixed by the **initial and final positions** alone. Its work does **not depend on the path** taken between them.

EXAMPLES

Gravitational force: its work depends only on the change in height

Conservative ✓

Spring (elastic) force: its work depends only on the stretch or compression

Conservative ✓

PROPERTIES

i End Points Only

The work done **by or against** the force is decided by the **starting and ending positions** alone.

ii Path Independent

The work does **not depend on the path** that the body follows between the two positions.

iii Closed Path

For a **closed path** (round trip), the total work done is **zero**.

Take gravity as an example. Gravity does work W_{PQ} from P to Q , and W_{QP} from Q to P . These two are equal in size but opposite in sign. So for the round trip $P \rightarrow Q \rightarrow P$:

NET WORK OVER A CLOSED PATH

$$W_{PQ} = -W_{QP} \Rightarrow W_{PQ} + W_{QP} = 0 \Rightarrow \oint \vec{F} \cdot d\vec{s} = 0$$

1.8 Non-conservative Forces

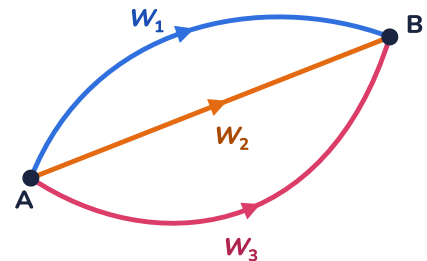
DEFINITION

A **non-conservative force** is a force whose work **depends on the path** followed between two positions.

Move a body from A to B along **three different paths**. Let the work done be W_1 , W_2 and W_3 . For a non-conservative force, these values are **not equal**:

$$W_1 \neq W_2 \neq W_3$$

So for a **closed path** $A \rightarrow B \rightarrow A$, the total work is **not zero**. The lost work turns into **heat**.



Different paths give different work (path dependent)

SOME EXAMPLES

1 Viscous force

2 Tension

3 Force of friction

• WORK DONE BY FRICTION

Friction always opposes the **relative motion** between surfaces. Its work can be **negative**, **zero** or even **positive**.

I $F < f_s$

The push is **less than the static friction**. So the block **stays at rest** ($s = 0$).

$$W = 0$$

II $F > f_s$

The block **moves**. Kinetic friction acts against the displacement. So it does **negative work**.

$$W = -\mu_k M g s$$

III Block on Block

Block A rests on block B , and B is pulled. Friction does **positive work on A** and **negative work on B** .

$$W_A = +fs, \quad W_B = -fs$$

REMEMBER

Conservative vs Non-conservative

Round a **closed path**, a conservative force does **zero** net work ($\oint \vec{F} \cdot d\vec{s} = 0$). Then **mechanical energy is conserved**. A non-conservative force does **non-zero** work round the same path. Then mechanical energy is **not conserved**. Most of the lost energy becomes **heat**.

A moving body can do work, so it has **energy**. The energy of motion is called **kinetic energy**. The **work-energy theorem** connects the work done on a body with the change in its kinetic energy.

2.1 Energy

DEFINITION

Energy is the **capacity (ability) of a body to do work**.

Nature

Scalar quantity, like work. It has magnitude only, no direction.

Units

joule (SI) and **erg** (CGS).

Dimensions

$[ML^2T^{-2}]$, the same as work.

• PRACTICAL UNITS OF ENERGY

PRACTICAL UNIT OF ENERGY	SYMBOL	VALUE IN SI
Erg	erg	10^{-7} J
Electron volt	eV	1.6×10^{-19} J
Calorie	cal	4.186 J
Kilowatt hour	kWh	3.6×10^6 J

2.2 Kinetic Energy

DEFINITION

Kinetic energy is the energy that a body has **because of its motion**. It is equal to the work that the moving body can do before it comes to rest.

KINETIC ENERGY

$$KE = \frac{1}{2}mv^2$$

where, m = mass of the body, v = velocity of the body

EVERYDAY EXAMPLES

- ◆ A **bullet** from a gun goes through a target.
- ◆ A fast **stream of water** turns a water mill.

- ◆ Moving **air (wind)** turns a windmill.
- ◆ A **hammer** in motion pushes a nail into wood.

Never Negative

A moving body always has **positive** KE. KE can **never be negative**.

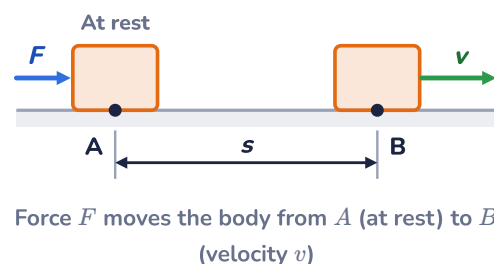
Depends on the Frame

The value of KE changes with the **frame of reference**. **e.g.** A person sitting in a moving train has KE when seen from the ground. Seen from the train, the person's KE is **zero**.

2.3 Measurement of Kinetic Energy

Take a body of mass m that is **at rest**. A force F acts on it and moves it along the direction of the force. The body speeds up and reaches a velocity v .

All the **work done** by the force gets stored in the body. This stored energy is its **kinetic energy**.



METHOD 1 • USING CALCULUS

DERIVATION

- 1 Let the force move the body through a small displacement ds along the force ($\theta = 0^\circ$). The small work done is:

$$dW = \vec{F} \cdot d\vec{s} = F ds \cos 0^\circ = F ds$$

- 2 By **Newton's second law**, the force gives the body an acceleration a . So $F = ma$:

$$dW = ma ds$$

- 3 Write the acceleration as $a = \frac{dv}{dt}$:

$$dW = m \frac{dv}{dt} ds$$

- 4 Rearrange the terms. Then use $\frac{ds}{dt} = v$, the velocity of the body:

$$dW = m ds \frac{dv}{dt} = m \frac{ds}{dt} dv = mv dv$$

- 5 Integrate from **rest (0)** to the final velocity v . This gives the total work:

$$W = \int_0^v mv \, dv = m \left[\frac{v^2}{2} \right]_0^v = \frac{1}{2}mv^2$$

RESULT

This work becomes the **kinetic energy** of the body:

$$\text{KE} = \frac{1}{2}mv^2$$

So the kinetic energy of a body is **half of its mass multiplied by the square of its velocity**.

• METHOD 2 • USING AN EQUATION OF MOTION

For a **constant force**, we get the same result without calculus.

DERIVATION

- 1 A constant force moves the body through a distance s . The work done is:

$$W = Fs = mas$$

- 2 The body starts from **rest** ($u = 0$) and reaches the speed v . From $v^2 = u^2 + 2as$:

$$v^2 = 2as \Rightarrow as = \frac{v^2}{2}$$

- 3 Put $as = \frac{v^2}{2}$ in $W = m(as)$. This gives the kinetic energy:

$$W = m(as) = m \cdot \frac{v^2}{2} = \frac{1}{2}mv^2$$

NOTE

How KE Depends on m and v

From $\text{KE} = \frac{1}{2}mv^2$, we get $\text{KE} \propto m$ and $\text{KE} \propto v^2$. So a **heavier** body or a **faster** body has more kinetic energy. KE depends on the **square** of the speed:

Speed is **doubled**

KE becomes 4 times

Speed is **tripled**

KE becomes 9 times

2.4 Kinetic Energy and Momentum

Kinetic energy and linear momentum both depend on **mass** and **velocity**. So we can link them by **removing** v .

DERIVATION

- 1 Linear momentum is $p = mv$, so $v = \frac{p}{m}$. Put this in the formula for kinetic energy:

$$\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}m \left(\frac{p}{m} \right)^2 = \frac{p^2}{2m}$$

- 2 Rearrange this to get momentum in terms of kinetic energy:

$$p^2 = 2m \text{KE} \Rightarrow p = \sqrt{2m \text{KE}}$$

KE FROM MOMENTUM

$$\text{KE} = \frac{p^2}{2m}$$

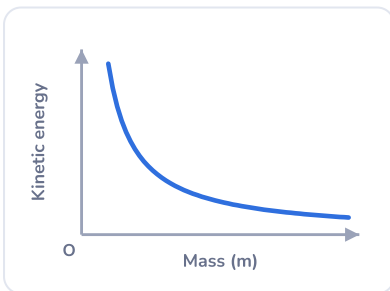
MOMENTUM FROM KE

$$p = \sqrt{2m \text{KE}}$$

This relation gives **three special cases**. Each case has its own graph.

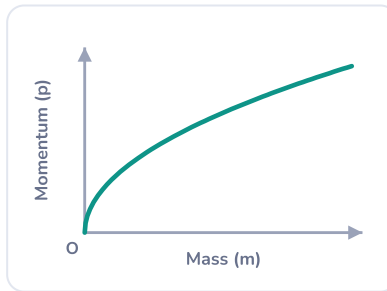
i p is constant

$\text{KE} \propto \frac{1}{m}$. For the same momentum, a **lighter body** has more KE.



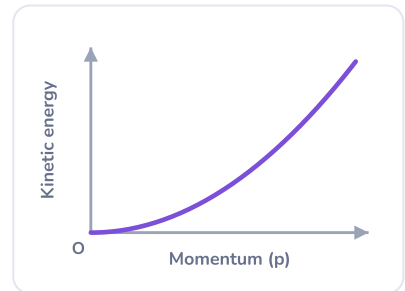
ii KE is constant

$p^2 \propto m$, so $p \propto \sqrt{m}$. The momentum **grows as** $\sqrt{m}$.



iii m is constant

$p^2 \propto \text{KE}$, so $p \propto \sqrt{\text{KE}}$. The KE **grows as** p^2 .



2.5 Work-Energy Theorem (Constant Force)

LAW

Work-Energy Theorem

The work done by the **net force** on a body equals the **change in its kinetic energy**. This is called the **work-energy theorem** or the **work-energy principle**.

Let a **constant force** F give a body an acceleration a . Over a distance s , its velocity changes from u to v .

DERIVATION

- 1 Use the equation of motion that links velocity and distance:

$$v^2 - u^2 = 2as$$

- 2 Multiply both sides by $\frac{1}{2}m$:

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mas$$

- 3 By Newton's second law, $F = ma$. So $mas = Fs = W$:

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = Fs = W$$

WORK-ENERGY THEOREM

$$W_{net} = K_f - K_i = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \Delta KE$$

where, K_f = final kinetic energy, K_i = initial kinetic energy of the body

Positive net work

KE increases · body speeds up

Negative net work

KE decreases · body slows down

Zero net work ($W_{net} = 0$)

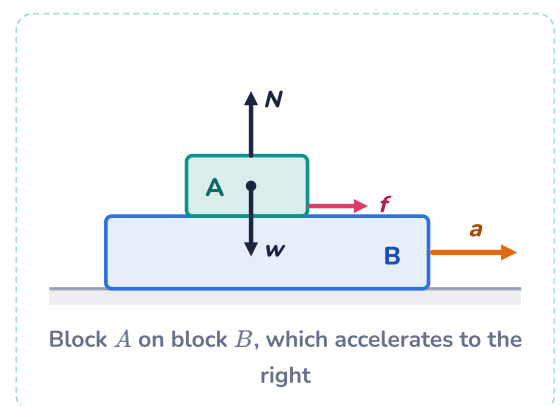
Speed does not change

2.6 A Block over Another Block

Block A rests on block B , and B is **accelerating**. We must use the work-energy theorem carefully here. The example also proves a useful point. The theorem is true in **every frame of reference**, for **every kind of force**.

Block B **accelerates to the right** with respect to the ground. Block A sits on it. The **weight** (w) and the **normal reaction** (N) on A are **perpendicular** to its displacement. So both do **zero work**.

Only **friction** can do work on A . In the frame of B , we must also count the work of the **pseudo force**.



i Ground Frame (Inertial)

No pseudo force acts here. Only friction does work on A :

$$W_{\text{friction}} = \Delta KE$$

A speeds up along with B . So the work done by friction on A is **positive**.

ii Frame of B (Non-inertial)

Here the pseudo force also acts on A . Apply the theorem to A :

$$W_{\text{friction}} + W_{\text{pseudo}} = \Delta KE$$

A is **at rest relative to B** . So $\Delta KE = 0$, which gives $W_{\text{pseudo}} = -W_{\text{friction}}$.

WORK-ENERGY THEOREM IN ANY FRAME

$$W_{\text{external}} + W_{\text{internal}} + W_{\text{pseudo}} + W_{\text{other}} = \Delta KE$$

The pseudo-force term is used only in a **non-inertial frame**. In an inertial frame, $W_{\text{pseudo}} = 0$.

NOTE

Key Point

The theorem is true for **every system of particles**. The forces may be conservative or non-conservative. When KE **rises**, the net work is positive. When KE **falls**, the net work is negative.

2.7 Work-Energy Theorem (Variable Force)

The theorem also holds when the force **changes**. We use very small quantities and then **integrate** along the path.

DERIVATION

- 1 A variable force F acts on a body of mass m . It moves the body through a small displacement ds along the force ($\theta = 0^\circ$). The small work done is:

$$dW = \vec{F} \cdot d\vec{s} = F ds \cos 0^\circ = F ds$$

- 2 The kinetic energy is $K = \frac{1}{2}mv^2$. Differentiate it with respect to time, keeping m constant. Use the chain rule on v^2 :

$$\frac{dK}{dt} = \frac{d}{dt} \left(\frac{1}{2}mv^2 \right) = \frac{1}{2}m \cdot 2v \cdot \frac{dv}{dt} = mv \frac{dv}{dt}$$

- 3 By **Newton's second law**, $m \frac{dv}{dt} = ma = F$. So:

$$\frac{dK}{dt} = \left(m \frac{dv}{dt} \right) v = (ma) v = Fv$$

4 Put $v = \frac{ds}{dt}$. Now KE changes at the same rate as the work is done. So for a small step, $dK = dW$:

$$\frac{dK}{dt} = F \frac{ds}{dt} \Rightarrow dK = F ds = dW$$

5 Integrate from the **initial state** (position x_i , KE K_i) to the **final state** (position x_f , KE K_f):

$$\int_{K_i}^{K_f} dK = \int_{x_i}^{x_f} F ds \Rightarrow K_f - K_i = \int_{x_i}^{x_f} F ds = W$$

This proves the work-energy theorem for a variable force.

WORK-ENERGY THEOREM (VARIABLE FORCE)

$$K_f - K_i = \int_{x_i}^{x_f} \vec{F} \cdot d\vec{s} = W$$

The work done by the net (variable) force equals the change in KE

NOTE

- ◆ The work-energy theorem is **not a separate law**. We get it from Newton's second law.
- ◆ Newton's second law in 2-D or 3-D is a **vector** equation. The work-energy theorem is a **scalar** equation.
- ◆ It can be used for a **system of many particles** too. It works whether the forces are conservative or not.

03 PART 3 Potential Energy & Conservation of Energy

A body can also store energy because of its **position** or its **shape**. This stored energy is called **potential energy**. When only conservative forces act, the sum of kinetic and potential energy **stays constant**.

3.1 Potential Energy

DEFINITION

The energy that a body has **because of its position or configuration** inside a field is called its **potential energy**. A change in configuration changes the potential energy too.

i Due to Position

An object on the **roof of a building** has potential energy. If it falls, it can **do work**.

ii Due to Configuration

A **stretched or compressed spring** has potential energy because its **shape is changed**.

Units and dimensions (same as kinetic energy)

joule (J) · [ML²T⁻²]

NOTE

Potential Energy Changes into Kinetic Energy

Potential energy is defined only for forces where the **work done against the force** is **stored as energy**. The energy is stored due to the body's **position or configuration**. When the body is set free, the stored energy **changes back into kinetic energy**. Its position or configuration changes at the same time.

3.2 Gravitational Potential Energy

DEFINITION

Gravitational potential energy is the energy that a body has **because of its height above the surface of the Earth**.

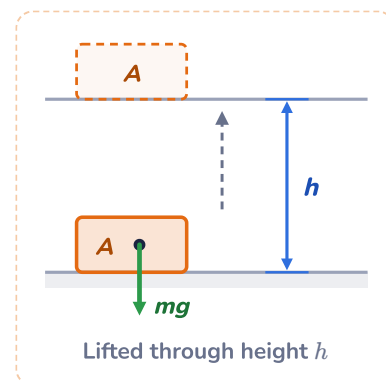
DERIVATION

- 1 Lift a body of mass m at **constant velocity**. There is no acceleration, so the lifting force balances gravity:

$$F = w = mg$$

- 2 Lift it through a height h . Take $h \ll R_E$, so g stays constant. The work done is stored as potential energy:

$$W = Fh = mgh = U$$



GRAVITATIONAL POTENTIAL ENERGY

$$U = mgh$$

where, m = mass, g = acceleration due to gravity, h = height measured from the reference level we choose

Force from PE

The force of gravity is the **negative gradient** of U :

$$F = -\frac{dU}{dh} = -mg$$

The minus sign shows that the force acts **downward**.

PE Changes into KE on Falling

Drop the body from rest at height h . It reaches the ground with speed $v = \sqrt{2gh}$:

$$\frac{1}{2}mv^2 = \frac{1}{2}m(2gh) = mgh$$

All the potential energy changes into **kinetic energy**.

3.3 Potential Energy of a Spring

DEFINITION

The energy stored in a **compressed or stretched** elastic spring is called its **potential energy**. Such a spring is usually **helical** or **spiral**. Take the spring as **massless**. Then all the work done to deform it is kept as **elastic potential energy**.

Take a **massless elastic spring**. One end is fixed to a **rigid support**. The other end is attached to a block of mass m on a **smooth horizontal surface**. Take the equilibrium position as $x = 0$. The spring can be in three states:

a Equilibrium

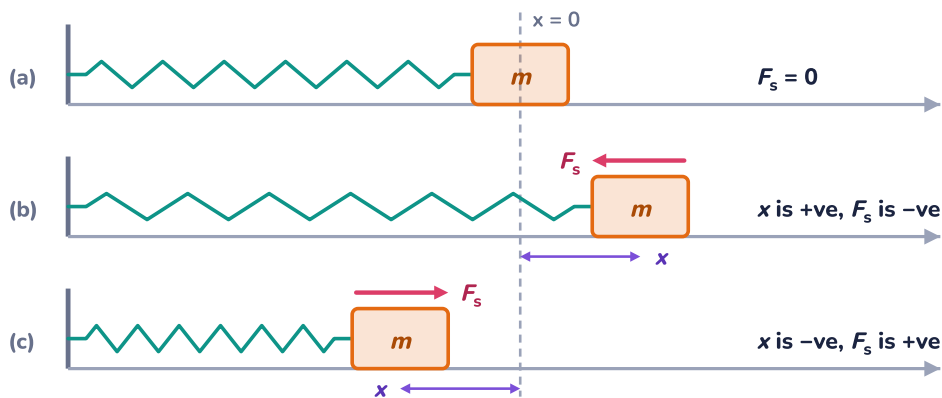
$x = 0$. The spring is relaxed, so $F_s = 0$.

b Stretched

$x > 0$. The restoring force **pulls the block back**. $F_s = -kx$ is negative.

c Compressed

$x < 0$. The restoring force **pushes the block out**. $F_s = -kx$ is positive.



(a) Equilibrium, (b) stretched and (c) compressed spring

HOOKE'S LAW • SPRING (RESTORING) FORCE

$$F_s = -kx$$

where, k = **spring constant** (SI unit N m^{-1}). The minus sign shows that the force acts **opposite** to the displacement x .

DERIVATION

1 In a tiny displacement dx , the spring force does work:

$$dW_s = F_s dx = -kx dx$$

2 Integrate as the block moves from x_i to x_f . The work is the **area under the F_s - x line**:

$$W_s = - \int_{x_i}^{x_f} kx dx = -k \left[\frac{x^2}{2} \right]_{x_i}^{x_f} = \frac{1}{2} kx_i^2 - \frac{1}{2} kx_f^2$$

From the Mean Position ($x_i = 0$)

Stretch or compress the spring from rest to x :

$$W_s = -\frac{1}{2} kx^2$$

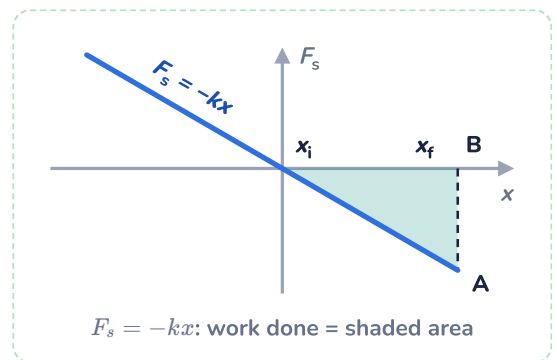
$$\text{PE stored} = -W_s = \frac{1}{2} kx^2.$$

Round Trip ($x_i \rightarrow x_f \rightarrow x_i$)

Pull the block to x_f and bring it back to x_i :

$$W_s = \frac{1}{2} kx_i^2 - \frac{1}{2} kx_i^2 = 0$$

The net work is **zero**. So the spring force is **conservative**.



ELASTIC POTENTIAL ENERGY OF A SPRING

$$U = \frac{1}{2} kx^2$$

where, x = stretch or compression of the spring from its natural length

NOTE

Conclusion

- ◆ The spring force depends on **position**: $F_s = -kx$ (Hooke's law).
- ◆ Its work depends only on the **initial and final positions**.
- ◆ Its work in a **complete cycle is zero**.
- ◆ Its work equals the **initial value minus the final value** of a potential energy function. So the spring force is a **conservative force**.

REMEMBER

Zero of Potential Energy

For a conservative force, the potential energy has **no fixed absolute value**. The reason is that the point of **zero potential energy** is our own **choice**. For the spring energy $\frac{1}{2}kx^2$, we take PE = 0 at the **equilibrium position** of the mass that oscillates.

3.4 Conservation of Mechanical Energy

PRINCIPLE

If only **conservative forces** act on a system, its **total mechanical energy** (kinetic + potential) **stays constant**. Energy only changes from KE to PE, or from PE to KE.

DERIVATION

- 1 For a **conservative force**, the work done equals the **negative of the change in PE**:

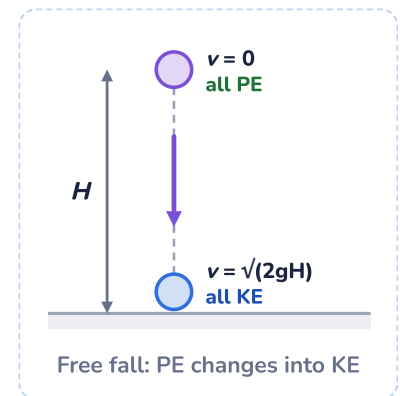
$$W = -\Delta U = U_i - U_f$$

- 2 By the **work-energy theorem**, the same work equals the change in KE:

$$W = \Delta KE = K_f - K_i$$

- 3 Equate the two and rearrange. The total energy does not change:

$$K_f - K_i = U_i - U_f \Rightarrow K_f + U_f = K_i + U_i$$



TOTAL MECHANICAL ENERGY IS CONSERVED

$$E = KE + PE = \frac{1}{2}mv^2 + U = \text{constant}$$

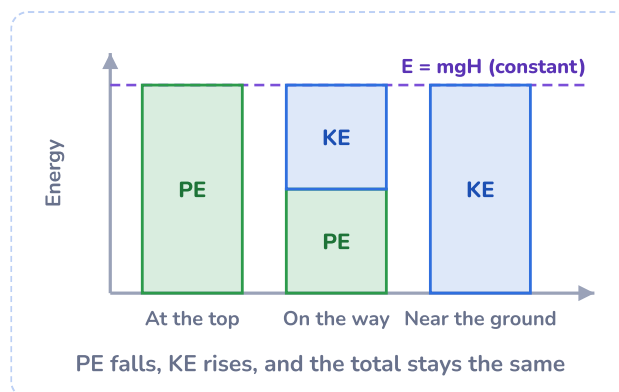
REMEMBER

Condition

Mechanical energy is conserved **only when non-conservative forces are absent**. Friction and air resistance are such forces. If they act, a part of the mechanical energy is **lost as heat**. Then KE + PE does not stay constant.

Take a body that **falls freely from a height H** . Its total energy is mgH at every point. As it falls, PE keeps changing into KE.

POSITION	KE	PE	TOTAL
At the top (released, $h = H$)	0	mgH	mgH
On the way (after falling x)	mgx	$mg(H - x)$	mgH
Just before the ground ($h = 0$)	mgH	0	mgH



3.5 Motion in a Vertical Circle

Tie a particle of mass m to a light string that cannot stretch (inextensible). Its length L is the radius R of the circle. The particle moves round a **vertical circle** with its centre at a fixed point O . At the lowest point A , it is given a horizontal speed u .

Take any point B at a height $h = R(1 - \cos \theta)$, where the speed is v . Resolve the weight mg into two parts. The **radial part** $mg \cos \theta$ acts along the string. The **tangential part** $mg \sin \theta$ acts along the circle. Two equations decide the motion.

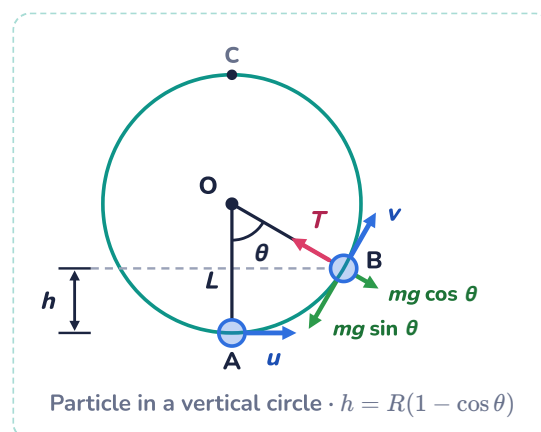
DERIVATION

- Conservation of energy** between A and B (the particle rises by h):

$$\frac{1}{2}m(u^2 - v^2) = mgh, \quad h = R(1 - \cos \theta)$$

- At B , the **centripetal force** comes from the tension T and the radial part $mg \cos \theta$. The tangential part $mg \sin \theta$ only changes the speed.

$$T - mg \cos \theta = \frac{mv^2}{R} \Rightarrow T = \frac{mv^2}{R} + mg \cos \theta$$



For the particle to **just complete the circle**, the minimum speeds and the tensions at three key points are:

POINT ON THE CIRCLE	MINIMUM SPEED	TENSION IN THE STRING
Highest point (C)	$v_{top} = \sqrt{gR}$	$T = 0$
String horizontal	$v = \sqrt{3gR}$	$T = 3mg$
Lowest point (A)	$u_{min} = \sqrt{5gR}$	$T = 6mg$

NOTE

The speed and the tension are **greatest at the lowest point**. They are **least at the highest point**. To complete the circle, the string must stay **tight** ($T \geq 0$) all the way round. The most critical point is the **top**.

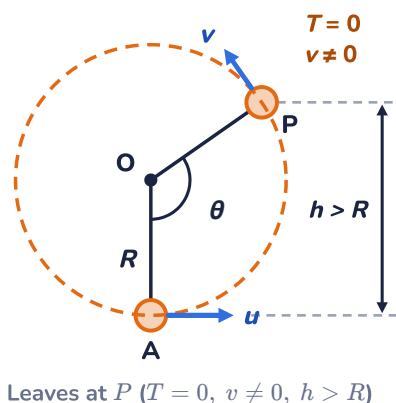
The speed u at the **lowest point A** decides the type of motion.

1 Completes the Circle • $u \geq \sqrt{5gR}$

The string stays **tight all the way**. The particle goes **over the top** with a speed of at least $v_{top} = \sqrt{gR}$.

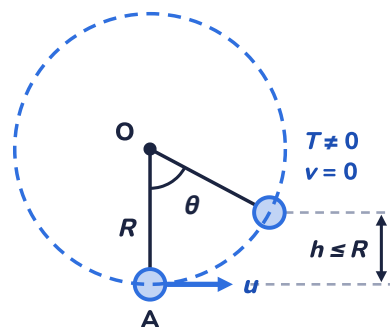
2 Leaves the Circle • $\sqrt{2gR} < u < \sqrt{5gR}$

The tension becomes **zero before the top**, where $90^\circ < \theta < 180^\circ$ and $h > R$. The particle **leaves the circle at P**. After that, it moves on a **projectile (parabolic) path**.



3 Oscillates • $0 < u \leq \sqrt{2gR}$

The speed becomes **zero in the lower half** ($0^\circ < \theta \leq 90^\circ$), while $T \neq 0$. It moves to and fro like a **pendulum**.



3.6 Various Forms of Energy

Energy exists in **many forms**. It can change from one form to another. The total amount of energy stays the same.

FORM OF ENERGY	WHAT IT IS
Mechanical energy	It is kinetic energy + potential energy . KE comes from motion. PE comes from position or configuration.
Internal energy	It is the sum of the KE and PE of all the molecules in a body. It comes from molecular motion and the forces between molecules.
Heat energy	It comes from the random (disordered) motion of molecules. It is related to internal energy. e.g. Rubbing our hands makes them warm.
Chemical energy	It is the difference in energy between a stable compound and the atoms that form it. Exothermic reactions give it out. Endothermic reactions take it in.

FORM OF ENERGY	WHAT IT IS
Electrical energy	It is carried by an electric current , which is a flow of charges in a field. We use it to light bulbs, run motors and work heaters.
Nuclear energy	Heavy nuclei can split (fission) . Light nuclei can join (fusion) . Both give out a huge amount of energy. Nuclear reactors and bombs use it.

3.7 Equivalence of Mass and Energy

CONCEPT

Einstein, 1905

Mass can be changed into **energy**, and energy can be changed into mass. So mass and energy are **equivalent**. They are not conserved separately. They are conserved together as one quantity, called **mass-energy**.

MASS-ENERGY EQUIVALENCE

$$E = mc^2$$

where, m = mass that disappears, E = energy that appears, c = speed of light in vacuum ($\approx 3 \times 10^8 \text{ m s}^{-1}$)

EXAMPLE

Suppose just **1 kg** of matter changes fully into energy:

$$E = 1 \times (3 \times 10^8)^2 = 9 \times 10^{16} \text{ J}$$

This is about the **yearly output** of a large power station of **3000 MW**.

3.8 Principle of Conservation of Energy

PRINCIPLE

Energy can **neither be created nor destroyed**. One form of energy can only **change into another form**. If some energy is lost in one form, the same amount is gained in another form. So the **total energy of an isolated system stays constant**.

NOTE

The whole **universe** is an isolated system. So the total energy of the universe never changes. When some part loses energy, some other part gains exactly that much. This principle **cannot be proved**. But it has **never been found to fail**.

04 PART 4 Power & Collisions

Power tells us how fast work is done. In a **collision**, two bodies push each other with large forces for a very short time. **Linear momentum is conserved** in every collision. Kinetic energy is conserved only in elastic collisions.

4.1 Power

DEFINITION

Power is the **rate of doing work** (or of transferring energy) by a person or a machine.

AVERAGE POWER

$$P_{av} = \frac{\text{Work done}}{\text{Time taken}} = \frac{W}{t}$$

INSTANTANEOUS POWER

$$P = \frac{dW}{dt} = \vec{F} \cdot \vec{v}$$

Make the time interval smaller and smaller ($\Delta t \rightarrow 0$). The average power then becomes the **instantaneous power**. We also use $dW = \vec{F} \cdot d\vec{x}$:

DERIVATION

- 1 Take the limit of the average power as $\Delta t \rightarrow 0$:

$$P = \lim_{\Delta t \rightarrow 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt}$$

- 2 Put $dW = \vec{F} \cdot d\vec{x}$ and use $\frac{d\vec{x}}{dt} = \vec{v}$:

$$P = \vec{F} \cdot \frac{d\vec{x}}{dt} = \vec{F} \cdot \vec{v}$$

• UNITS OF POWER

UNIT	SYMBOL	EQUIVALENT
Watt (SI unit of power)	W	1 J s^{-1}
Kilowatt	kW	10^3 W
Horsepower	hp	746 W
Kilowatt-hour (a unit of energy)	kWh	$3.6 \times 10^6 \text{ J}$

Nature

Scalar quantity. It is the ratio of two scalars, W and t .

Dimensions

$$[ML^2T^{-3}]$$

At an Instant

Power is the **dot product** of force and velocity, $\vec{F} \cdot \vec{v}$.

REMEMBER

Another name for the **kilowatt-hour (kWh)** is the **Board of Trade (BOT) unit**. It is the **commercial unit of electrical energy**. It is **not** a unit of power.

4.2 Collisions

DEFINITION

A **collision** is an **isolated event**. In it, two or more bodies exert **large forces** on each other for a **short time**. The bodies **need not touch** each other.

EXAMPLE

No Contact Needed

In **Rutherford's scattering experiment**, an α -particle does not touch the nucleus. It is turned away by **electrostatic repulsion**. Still, we say that the α -particle **collides** with the nucleus.

• TWO TYPES OF COLLISION

FEATURE	ELASTIC COLLISION	INELASTIC COLLISION
Kinetic energy	No KE is lost	Some KE is lost
Forces involved	Conservative	May be non-conservative
Energy change	KE does not change into heat, light or sound	A part of the KE changes into heat, light or sound
Examples	Sub-atomic particles, glass balls	Two vehicles; a ball and the floor

• CONSERVATION OF LINEAR MOMENTUM IN A COLLISION

DERIVATION

- 1 During the collision time Δt , the two bodies exert **impulsive forces** on each other. So their momenta change:

$$\Delta p_1 = F_{12} \Delta t, \quad \Delta p_2 = F_{21} \Delta t$$

- 2 By **Newton's third law**, $F_{12} = -F_{21}$. So the two changes cancel:

$$\Delta p_1 + \Delta p_2 = 0$$

3 So the total momentum does not change:

$$\Delta(p_1 + p_2) = 0 \Rightarrow p_1 + p_2 = \text{constant}$$

MOMENTUM CONSERVED IN EVERY COLLISION

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

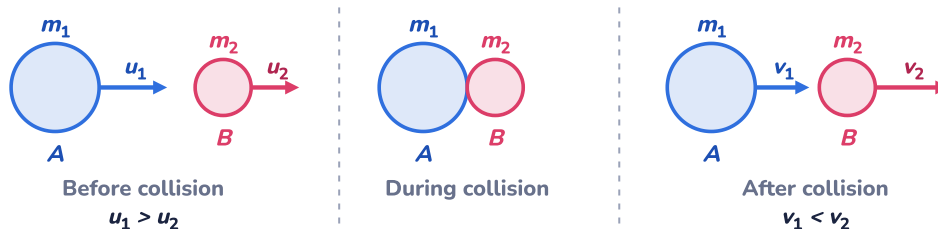
The total linear momentum is conserved at **every instant** of the collision. This holds in elastic as well as inelastic collisions.

4.3 Elastic Collision in One Dimension

DEFINITION

In an **elastic collision in one dimension**, two bodies first move along the **same straight line**. They collide with **no loss of kinetic energy**. After the collision, they keep moving along the **same line**.

Two bodies A and B have masses m_1 and m_2 . They move along the same line with velocities u_1 and u_2 , where $u_1 > u_2$. After they collide, both still move the same way. Their velocities are now v_1 and v_2 , with $v_2 > v_1$. In this collision, **momentum** and **kinetic energy** are both conserved.



The two bodies before, during and after an elastic collision

DERIVATION

1 **Momentum is conserved.** Bring the terms of each mass to one side:

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2 \quad \dots (i)$$

$$m_1 u_1 - m_1 v_1 = m_2 v_2 - m_2 u_2$$

$$m_1(u_1 - v_1) = m_2(v_2 - u_2) \quad \dots (ii)$$

- 2 **Kinetic energy is conserved** (elastic collision). Cancel the $\frac{1}{2}$ and group the terms of each mass. Then use $a^2 - b^2 = (a + b)(a - b)$:

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$m_1u_1^2 - m_1v_1^2 = m_2v_2^2 - m_2u_2^2$$

$$m_1(u_1 + v_1)(u_1 - v_1) = m_2(v_2 - u_2)(v_2 + u_2) \quad \dots (iii)$$

- 3 **Divide (iii) by (ii).** The factors $m_1(u_1 - v_1)$ and $m_2(v_2 - u_2)$ cancel out:

$$\frac{m_1(u_1 + v_1)(u_1 - v_1)}{m_1(u_1 - v_1)} = \frac{m_2(v_2 - u_2)(v_2 + u_2)}{m_2(v_2 - u_2)} \Rightarrow u_1 + v_1 = v_2 + u_2$$

- 4 Rearrange to get an important result:

$$u_1 - u_2 = v_2 - v_1 \quad \dots (iv)$$

APPROACH VELOCITY = SEPARATION VELOCITY

$$u_1 - u_2 = v_2 - v_1$$

The speed with which the bodies **move apart** after the collision equals the speed with which they **came together** before it.

• VELOCITIES AFTER THE COLLISION

Now solve the relative velocity relation (iv) together with the momentum equation (i). This gives v_1 and v_2 using only u_1 , u_2 and the masses.

DERIVATION

- 1 From $u_1 - u_2 = v_2 - v_1$, write v_2 in terms of v_1 :

$$v_2 = u_1 - u_2 + v_1 \quad \dots (v)$$

- 2 Put (v) in the momentum equation (i). Then expand the right-hand side:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2(u_1 - u_2 + v_1)$$

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2u_1 - m_2u_2 + m_2v_1$$

- 3 Collect the v_1 terms on one side and the u terms on the other:

$$(m_1 - m_2)u_1 + 2m_2u_2 = (m_1 + m_2)v_1$$

4 Solve for v_1 . This is Eq. (vi). Put it back in (v) to get v_2 , which is Eq. (vii):

$$v_1 = \frac{(m_1 - m_2)u_1 + 2m_2u_2}{m_1 + m_2} \quad \dots (vi)$$

VELOCITY OF BODY A • (VI)

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) u_1 + \left(\frac{2m_2}{m_1 + m_2} \right) u_2$$

VELOCITY OF BODY B • (VII)

$$v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2} \right) u_2 + \left(\frac{2m_1}{m_1 + m_2} \right) u_1$$

So (vi) and (vii) tell us the velocities **after** the collision when the velocities **before** it are known. Three special cases are important.

I Equal Masses ($m_1 = m_2 = m$)

Put $m_1 = m_2 = m$ in Eqs. (vi) and (vii). The terms with $(m_1 - m_2)$ become zero.

The two bodies simply **exchange their velocities**.

$$\begin{aligned} v_1 &= \frac{2mu_2}{2m} = u_2 \\ v_2 &= \frac{2mu_1}{2m} = u_1 \end{aligned}$$

II Light Body Hits a Heavy Body at Rest ($m_1 \ll m_2, u_2 = 0$)

Put $u_2 = 0$ in Eqs. (vi) and (vii). Neglect m_1 in comparison with m_2 .

The light body **bounces back with the same speed**. The heavy body stays almost at rest.

$$\begin{aligned} v_1 &= \frac{-m_2u_1}{m_2} = -u_1 \\ v_2 &\approx 0 \end{aligned}$$

III Heavy Body Hits a Light Body at Rest ($m_1 \gg m_2, u_2 = 0$)

Put $u_2 = 0$ in Eqs. (vi) and (vii). Neglect m_2 in comparison with m_1 .

The heavy body moves on **almost unchanged**. The light body flies off at about **twice that speed**, $2u_1$.

$$\begin{aligned} v_1 &\approx \frac{m_1u_1}{m_1} = u_1 \\ v_2 &\approx \frac{2m_1u_1}{m_1} = 2u_1 \end{aligned}$$

DEFINITION

Head-on (1-D) Collision

A collision is called **head-on** or **one-dimensional** when both bodies move along **one straight line**, before and after the collision.

REMEMBER

During and After the Collision

In an elastic collision, KE is conserved **only after the collision is over**. It is **not conserved during** the short time of the actual collision. At the moment of impact, the two bodies are **deformed**. They may even be **at rest relative to each other** for an instant. At that moment, the KE is held for a while as **elastic potential energy** of the squeezed bodies.

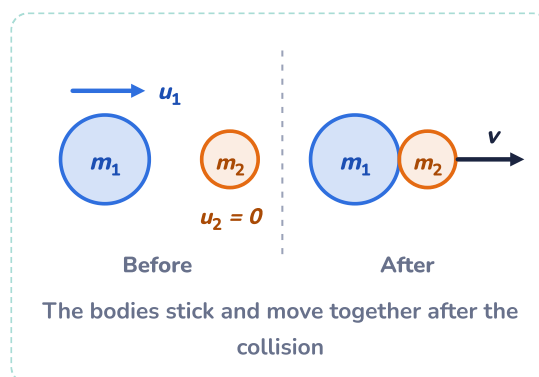
4.4 Perfectly Inelastic Collision

DEFINITION

A collision is **perfectly inelastic** if the two bodies **stick together** after colliding. They then move as **one body** with a **common velocity**.

A body of mass m_1 moving with velocity u_1 hits a body of mass m_2 moving with velocity u_2 along the same line. They join and move together with a common velocity v . Only **momentum** is conserved:

$$m_1 u_1 + m_2 u_2 = (m_1 + m_2) v$$



COMMON VELOCITY AFTER THE COLLISION

$$v = \frac{m_1 u_1 + m_2 u_2}{m_1 + m_2}$$

If the second body is at rest at first ($u_2 = 0$), then $v = \frac{m_1 u_1}{m_1 + m_2}$.

Momentum is conserved, but **kinetic energy is not**. Take $u_2 = 0$ and find the loss of kinetic energy.

DERIVATION

- 1 Find the total KE **before** the collision and **after** it:

$$E_1 = \frac{1}{2} m_1 u_1^2 \quad \dots (i)$$

$$E_2 = \frac{1}{2} (m_1 + m_2) v^2 \quad \dots (ii)$$

- 2 Put $v = \frac{m_1 u_1}{m_1 + m_2}$ in Eq. (ii):

$$E_2 = \frac{1}{2} (m_1 + m_2) \left[\frac{m_1 u_1}{m_1 + m_2} \right]^2 = \frac{1}{2} \frac{m_1^2 u_1^2}{m_1 + m_2}$$

3 The loss of kinetic energy is $E_1 - E_2$:

$$\Delta KE = E_1 - E_2 = \frac{1}{2}m_1u_1^2 - \frac{m_1^2u_1^2}{2(m_1 + m_2)}$$

4 Take $2(m_1 + m_2)$ as the common denominator and simplify the numerator:

$$\Delta KE = \frac{m_1^2u_1^2 + m_1m_2u_1^2 - m_1^2u_1^2}{2(m_1 + m_2)} = \frac{m_1m_2u_1^2}{2(m_1 + m_2)}$$

LOSS OF KINETIC ENERGY

$$\Delta KE = \frac{m_1m_2u_1^2}{2(m_1 + m_2)}$$

This is always **positive**. The lost KE appears mainly as **light, sound and heat**.

NOTE

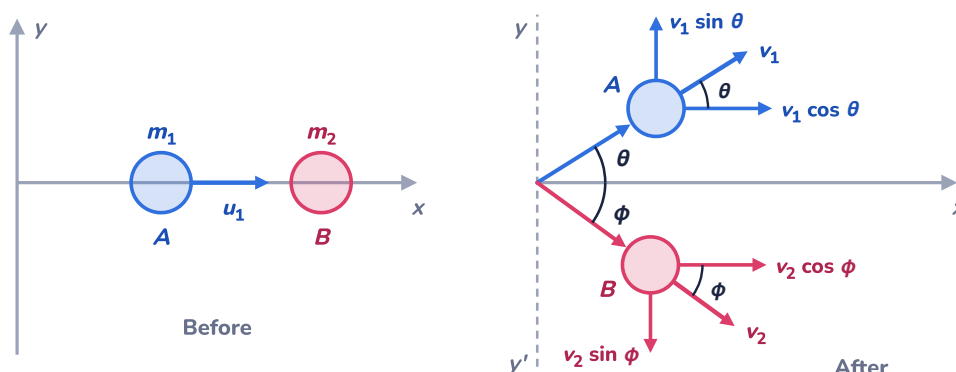
This collision causes the **greatest loss of kinetic energy** that is possible while momentum is conserved.

4.5 Collision in Two Dimensions

DEFINITION

A collision is **oblique** (or **two-dimensional**) if it is **not head-on**. Here the force between the bodies is not along the line of the initial velocity. So after the collision, the bodies move along **different lines**.

Body A (mass m_1) moves with speed u_1 along the x -axis. It hits body B (mass m_2), which is at rest. After the collision, A moves at an angle θ and B at an angle ϕ to the x -axis. Their speeds v_1 and v_2 are resolved into components along the axes.



Before: A moves towards B at rest · After: A moves at θ and B at ϕ to the x -axis

Governing Equations

$$\text{Momentum along } x: m_1 u_1 = m_1 v_1 \cos \theta + m_2 v_2 \cos \phi \quad \dots (i)$$

$$\text{Momentum along } y: 0 = m_1 v_1 \sin \theta - m_2 v_2 \sin \phi \quad \dots (ii)$$

$$\text{KE (if elastic): } m_1 u_1^2 = m_1 v_1^2 + m_2 v_2^2 \quad \dots (iii)$$

REMEMBER

Four Unknowns, Three Equations

We have **four unknowns**: v_1 , v_2 , θ and ϕ . But there are only **three equations**. So one quantity, usually the angle θ , must be **measured in an experiment**. Then the others can be found.

The three equations lead to **three special cases**.

I Glancing Collision

Here the incident particle **loses no kinetic energy**. It goes on almost **undeflected**. Here $\theta = 0^\circ$ and $\phi = 90^\circ$.

$$\text{From Eqs. (i) and (ii): } u_1 = v_1, \quad v_2 = 0$$

So the target particle gets **no kinetic energy**: $\frac{1}{2} m_2 v_2^2 = 0$.

II Head-on Collision

The incident particle keeps moving along its **original line**, so $\theta = 0^\circ$. The momentum equations become:

$$m_1 u_1 = m_1 v_1 + m_2 v_2 \cos \phi, \quad 0 = m_2 v_2 \sin \phi$$

So $\phi = 0^\circ$, and B also moves **on that line**. Equation (iii) stays the same. The problem becomes a **one-dimensional (head-on) collision**.

III Elastic Collision of Two Identical Particles

Let $m_1 = m_2$ and the incident speed $u_1 = u$. The target is at rest. The three equations become:

$$v_1^2 + v_2^2 = u^2 \quad \dots (iv)$$

$$v_1 \cos \theta + v_2 \cos \phi = u \quad \dots (v)$$

$$v_1 \sin \theta - v_2 \sin \phi = 0 \quad \dots (vi)$$

DERIVATION

1 Square Eq. (v). Both (iv) and (v) give u^2 , so equate them:

$$v_1^2 + v_2^2 = (v_1 \cos \theta + v_2 \cos \phi)^2 = v_1^2 \cos^2 \theta + v_2^2 \cos^2 \phi + 2v_1 v_2 \cos \theta \cos \phi$$

2 Move the cosine terms to the left side. Use $1 - \cos^2 = \sin^2$:

$$v_1^2 \sin^2 \theta + v_2^2 \sin^2 \phi = 2v_1 v_2 \cos \theta \cos \phi \quad \dots (vii)$$

3 From Eq. (vi), $v_2 \sin \phi = v_1 \sin \theta$. So $v_2^2 \sin^2 \phi = v_1^2 \sin^2 \theta$. Put this in (vii):

$$2v_1^2 \sin^2 \theta = 2v_1 v_2 \cos \theta \cos \phi \Rightarrow \cos \theta \cos \phi = \frac{v_1 \sin^2 \theta}{v_2} = \sin \theta \sin \phi$$

4 So $\cos \theta \cos \phi - \sin \theta \sin \phi = 0$, that is, $\cos(\theta + \phi) = 0$:

$$\cos(\theta + \phi) = 0 \Rightarrow \theta + \phi = 90^\circ$$

IDENTICAL PARTICLES • RIGHT ANGLE

$$\theta + \phi = 90^\circ$$

A moving particle hits an **identical particle at rest** in an **elastic** 2-D collision. Then, after the collision, their paths are **perpendicular** (at 90°) to each other.

NOTE

Inelastic Collision in 2-D

In an **inelastic** oblique collision, KE is **not conserved**. We can use only the momentum equations along x and y . So we get **three** equations for an elastic 2-D collision and only **two** for an inelastic one.

4.6 Coefficient of Restitution

DEFINITION

The **coefficient of restitution** (e) is the **relative velocity of separation** divided by the **relative velocity of approach**. Separation is measured after the collision, and approach before it. Another name for e is the **coefficient of resilience**.

COEFFICIENT OF RESTITUTION

$$e = \frac{\text{Relative velocity of separation}}{\text{Relative velocity of approach}} = \frac{v_2 - v_1}{u_1 - u_2}$$

where, u_1, u_2 = velocities before the collision, v_1, v_2 = velocities after it. The value of e is decided by the **materials** of the two bodies.

$$e = 1$$

Perfectly elastic. The velocity of separation is equal to the velocity of approach.

$$0 < e < 1$$

Inelastic (real) collision.

e.g. Glass balls: $e \approx 0.95$.
Lead balls: $e \approx 0.20$.

$$e = 0$$

Perfectly inelastic. The bodies stick together and do not separate.

● REBOUNTING OF A BALL

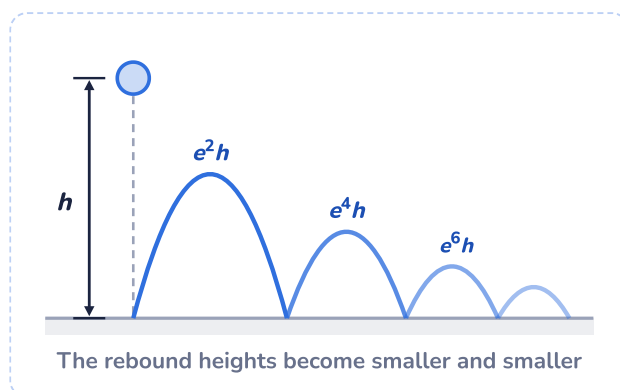
A ball is dropped from a height h . After each bounce, it rises to a **smaller height**. These two results are useful:

HEIGHT AFTER N REBOUNDS

$$h_n = e^{2n} h$$

TOTAL DISTANCE BEFORE STOPPING

$$s = h \left(\frac{1 + e^2}{1 - e^2} \right)$$



● DIFFERENT TYPES OF COLLISIONS

COLLISION	KINETIC ENERGY	COEFFICIENT (e)	MAIN DOMAIN
Elastic	Conserved	$e = 1$	Between atomic and sub-atomic particles
Inelastic	Not conserved	$0 < e < 1$	Between ordinary objects
Perfectly inelastic	Maximum loss of KE	$e = 0$	Bodies stick together
Super-elastic	KE increases	$e > 1$	In explosions

END OF CHAPTER 5 • NOTESSHALA.IN